Intuitive and Efficient Roof Modeling for Reconstruction and Synthesis

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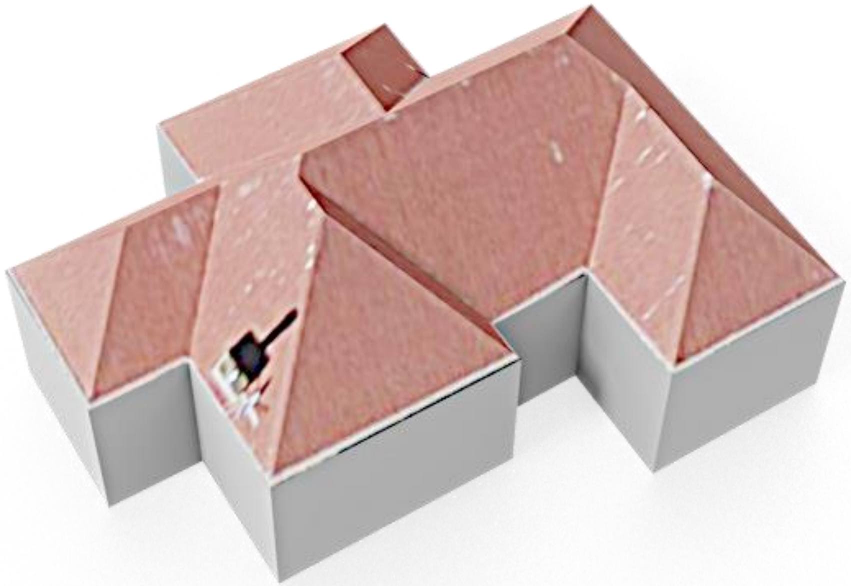
Introduction

Problem Formulation

Input

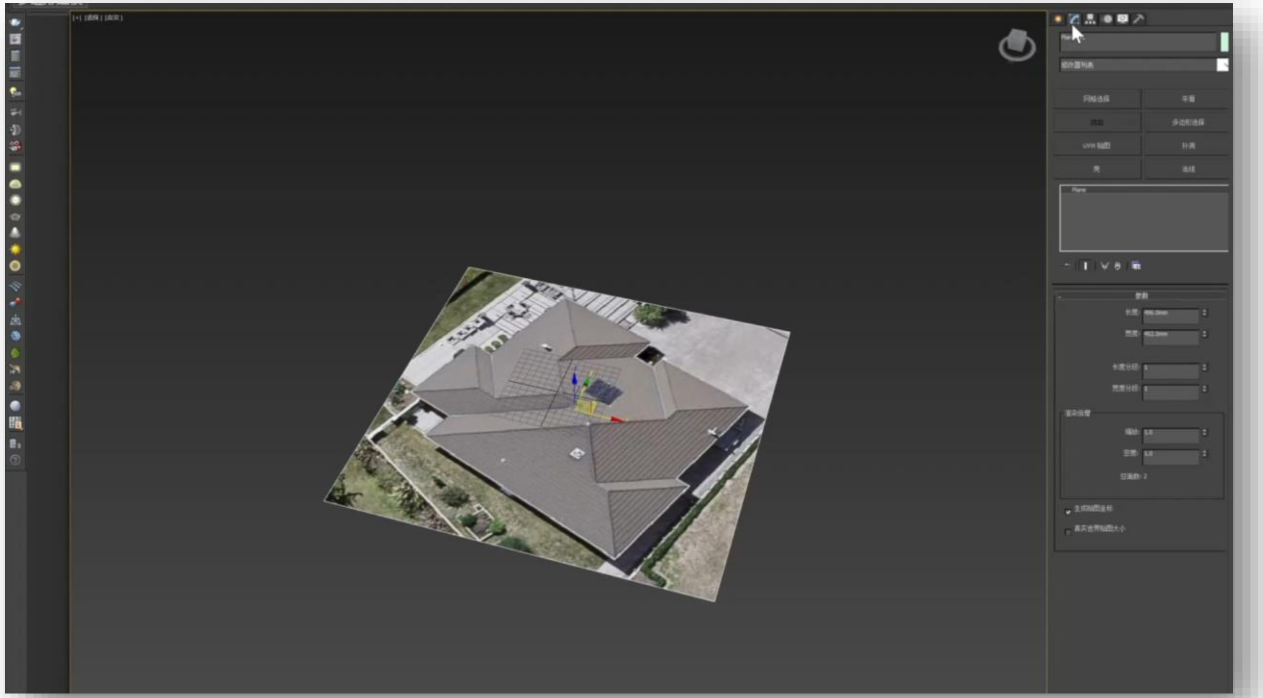


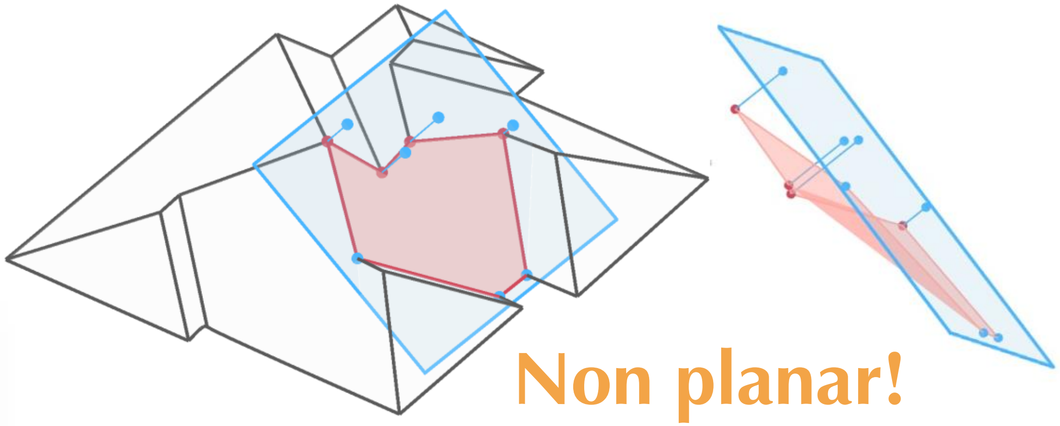
Output

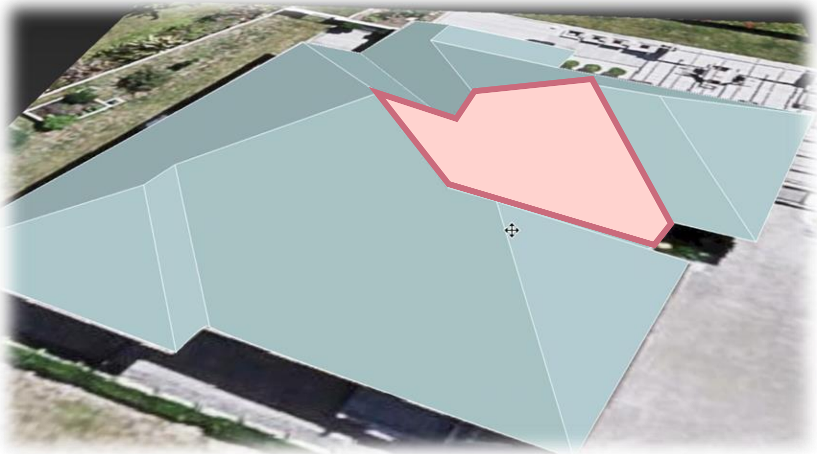
Existing Methods

Commercial Software: 3ds Max

Modeling time 3ds Max: **7**min, Ours: **2**min







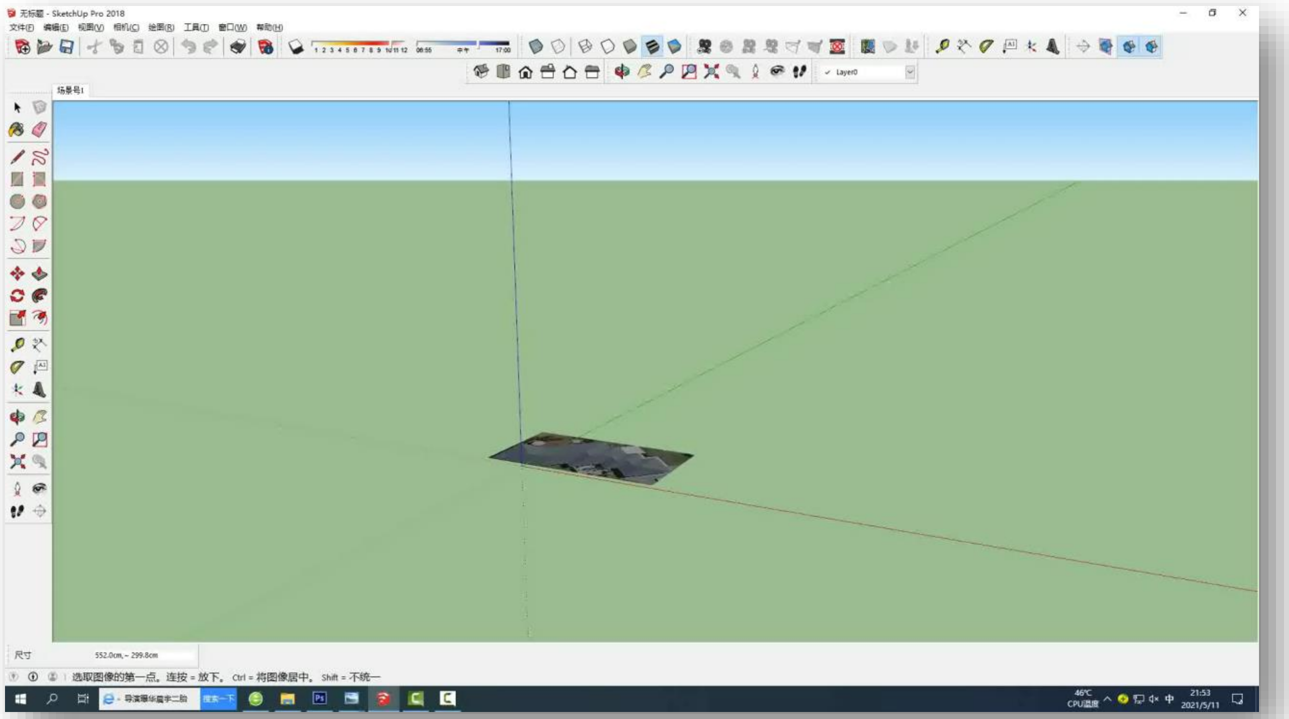
Modeling Operations

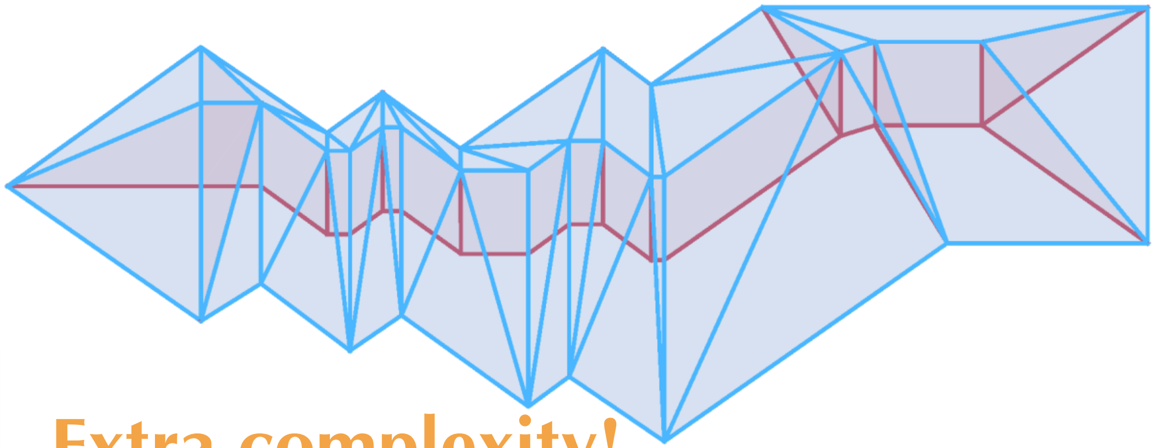
- Add vertex
- Add face
- Cut faces
- Move vertex

Existing Methods

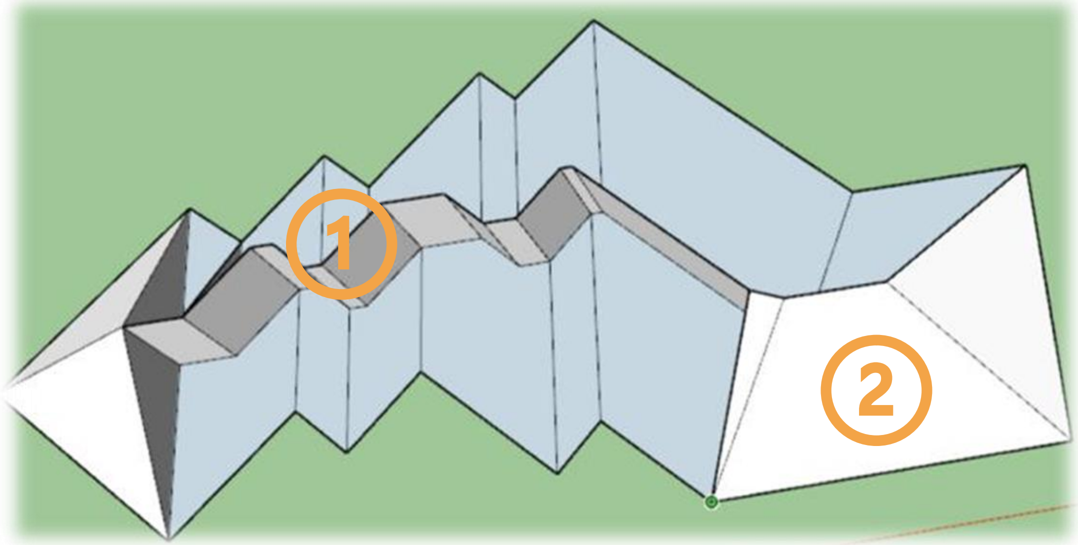
Commercial Software: SketchUp

Modeling time SketchUp: **25**min, Ours: **2**min





Extra complexity!

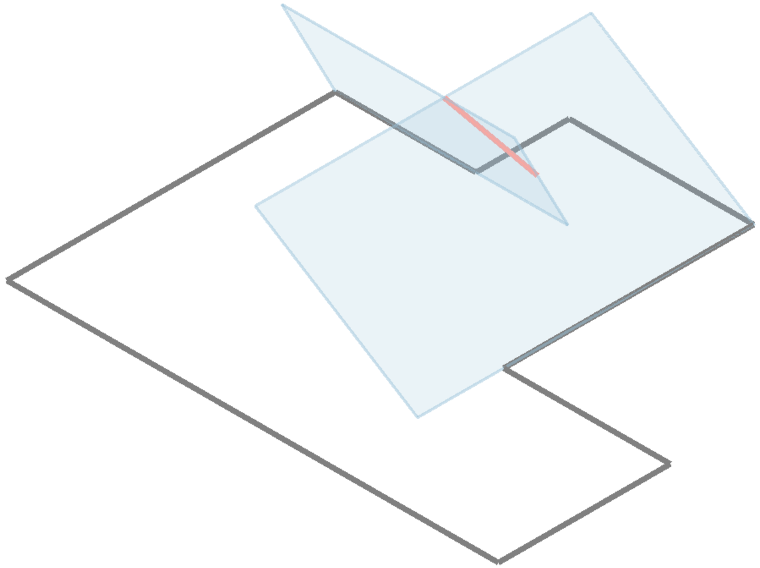


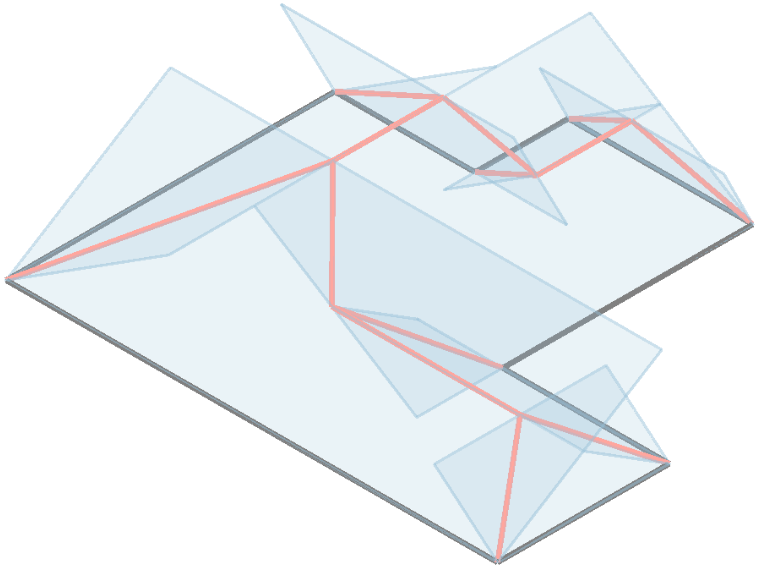
Modeling Operations

1. Build roof beams
2. Add planar roof tops

Existing Methods

Straight Skeleton Based Methods





Input 2D roof outline **Output** a roof embedded in 2D

Method solve for the roof topology & roof embedding at the same time

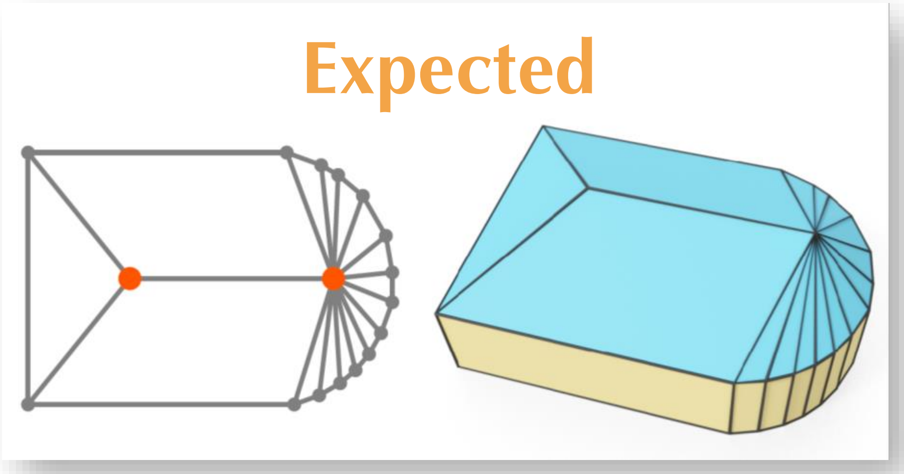
- Blue planes: stemming from the outlined edges
- Red structure: formed by intersection among blue planes

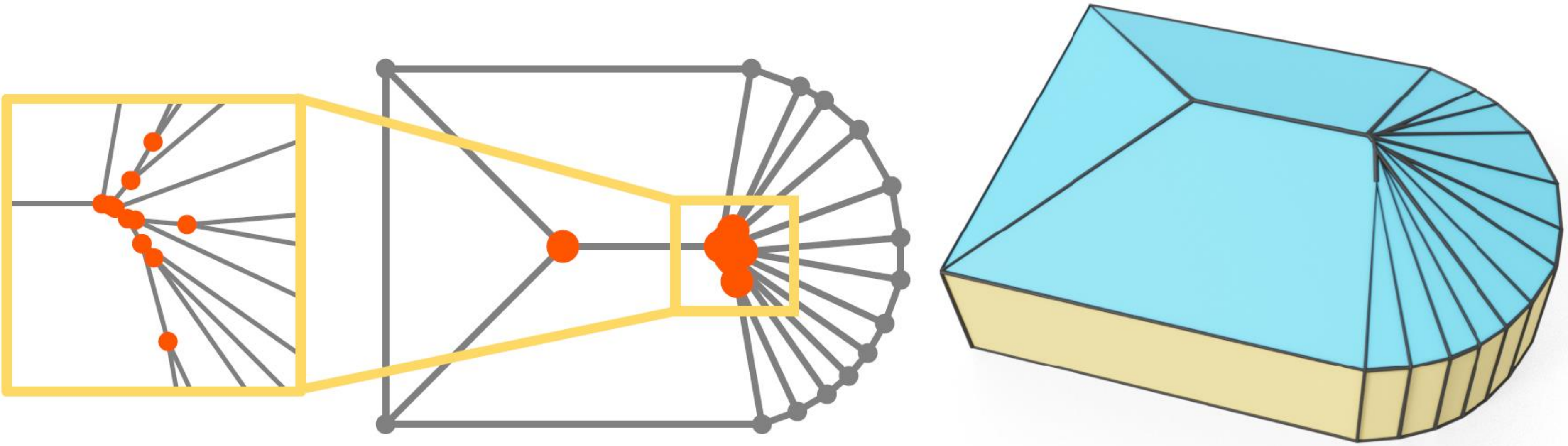
Reference:

1. Aichholzer and Aurenhammer, "**Straight Skeletons for general polygonal figures in the plane**", 1996
2. Eppstein and Erickson, "**Raising roofs, crashing cycles, and playing pool: Applications of a data structure for finding pairwise interactions**", 1999

Existing Methods

Straight Skeleton Based Methods

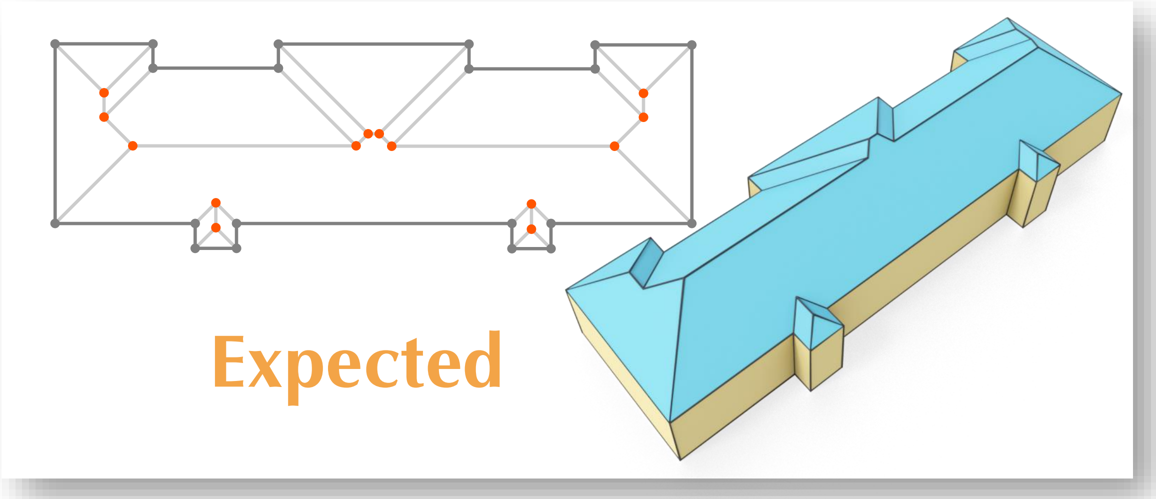


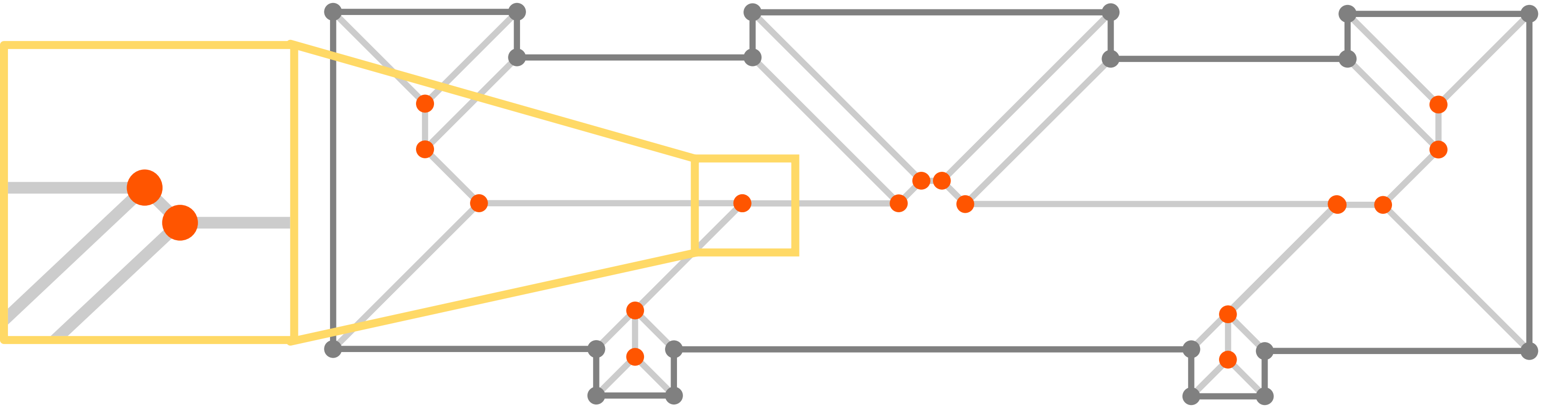


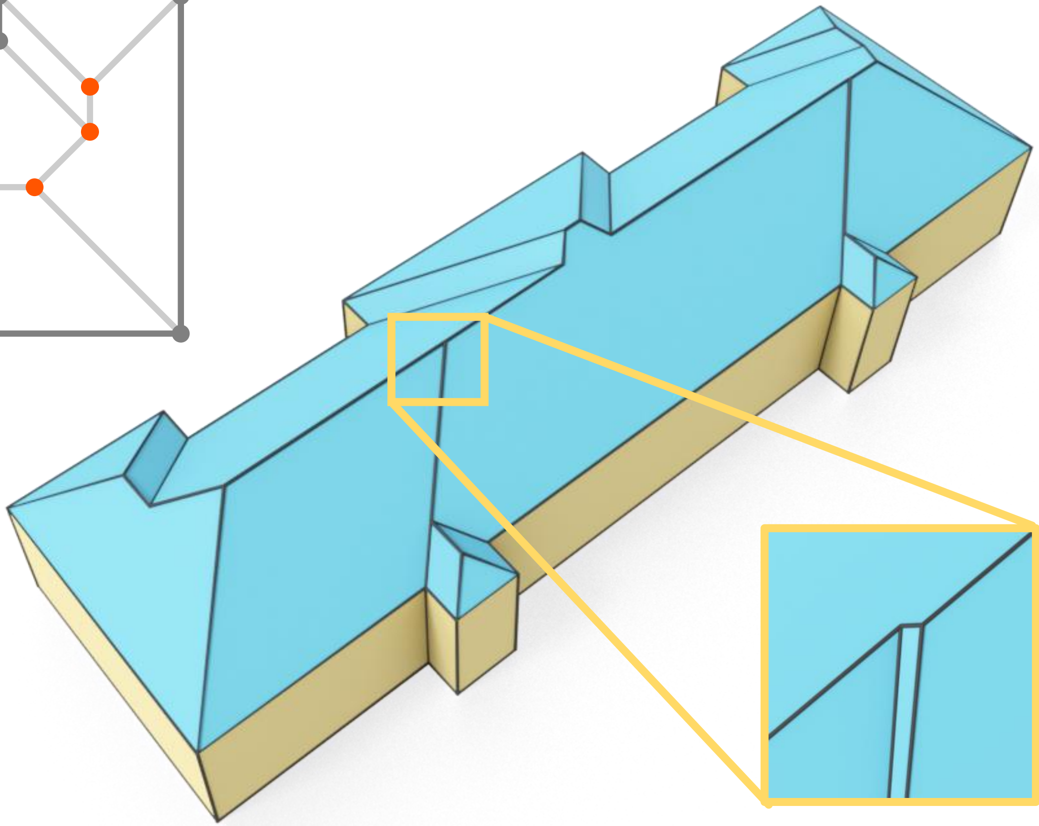
Problems wrong (and much more complicated) roof topology with spurious additional vertices

Existing Methods

Straight Skeleton Based Methods







Problems cannot handle the case where a face contains multiple outline edges

Existing Methods

	Commercial software	Straight skeleton	Our goal
Topologically correct	☺	☹	☺
Interactive editing	☺	☹	☺
Robust to noisy input	☺	☹	☺
Light user input	☹	☺	☺
Efficient for roof construction	☹	☺	☺
Easy to use for beginners	☹	☺	☺

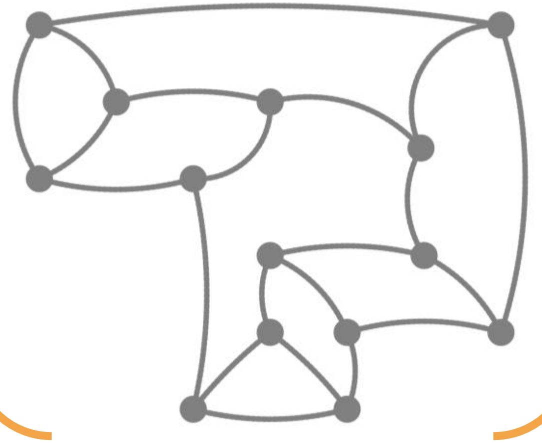
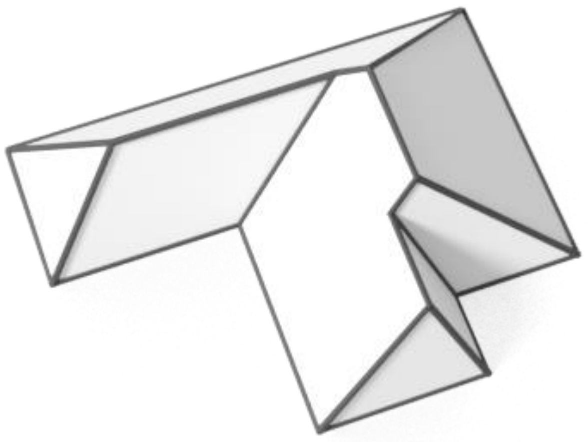
Methodology

Overview

Q1 How to **model** a roof?

Q2 How to describe whether a 3D roof is **planar**?

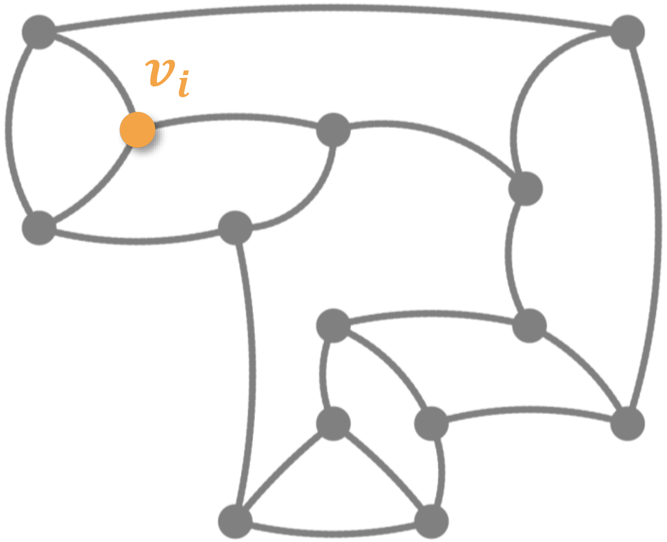
Q3 How to design **different regularizers** for different use cases?

$\underset{X}{\text{argmin}}$ **Roof Planarity** () **+ Regularizers** = 

Methodology

Notations

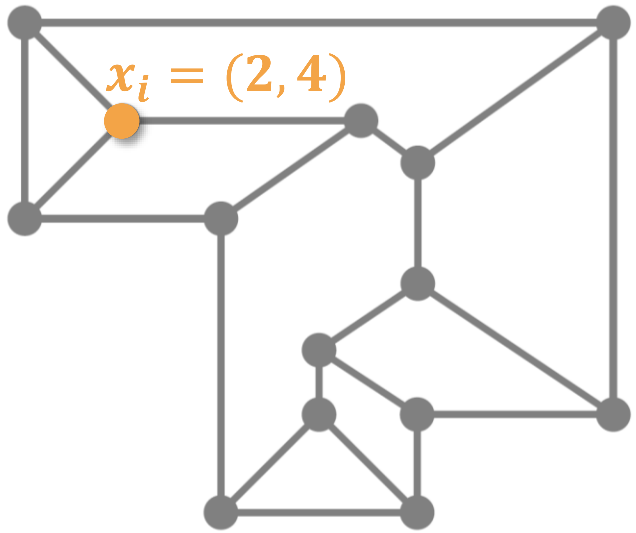
Roof Topology



Graph (V, F)

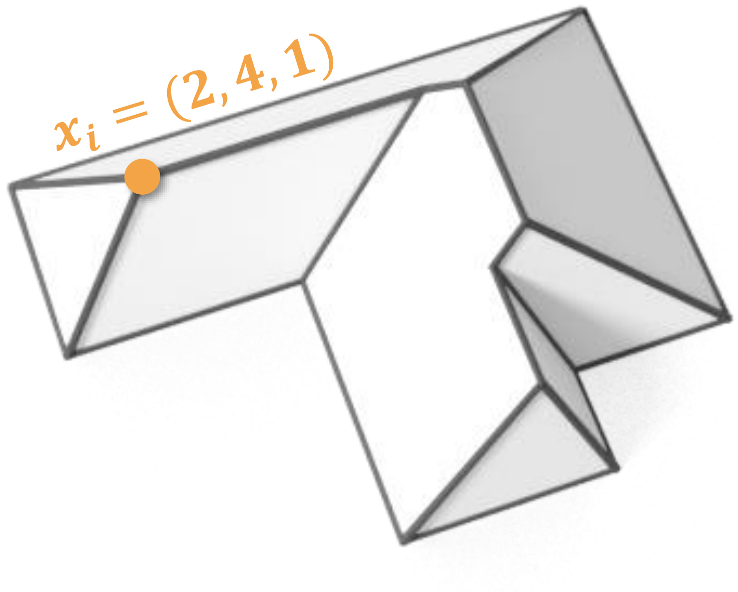
$|V| = n$

Roof Embedding



2D embedding:

$(V, F, X \in R^{n \times 2})$



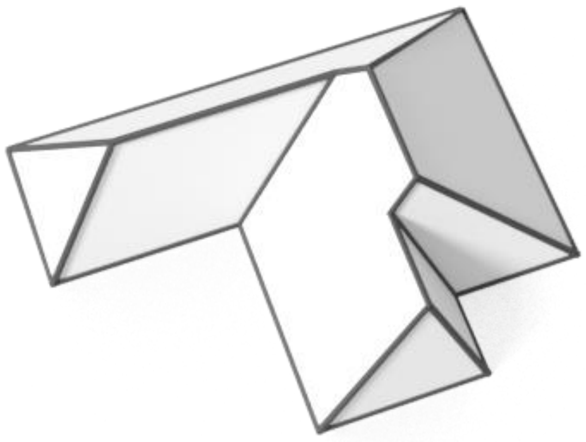
3D embedding:

$(V, F, X \in R^{n \times 3})$

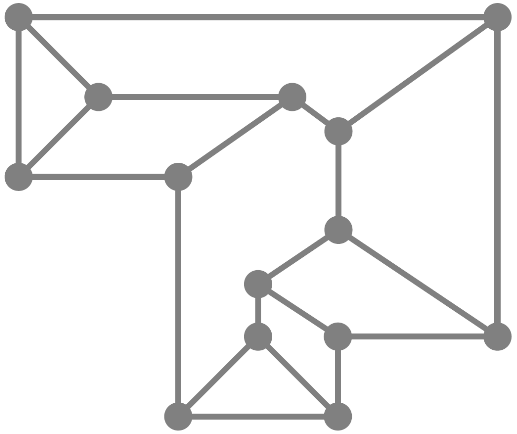
Methodology

Notations

Definition 1 We call a 3D embedding of a roof graph **valid** if each 3D roof face is planar and the roof has non-zero height.

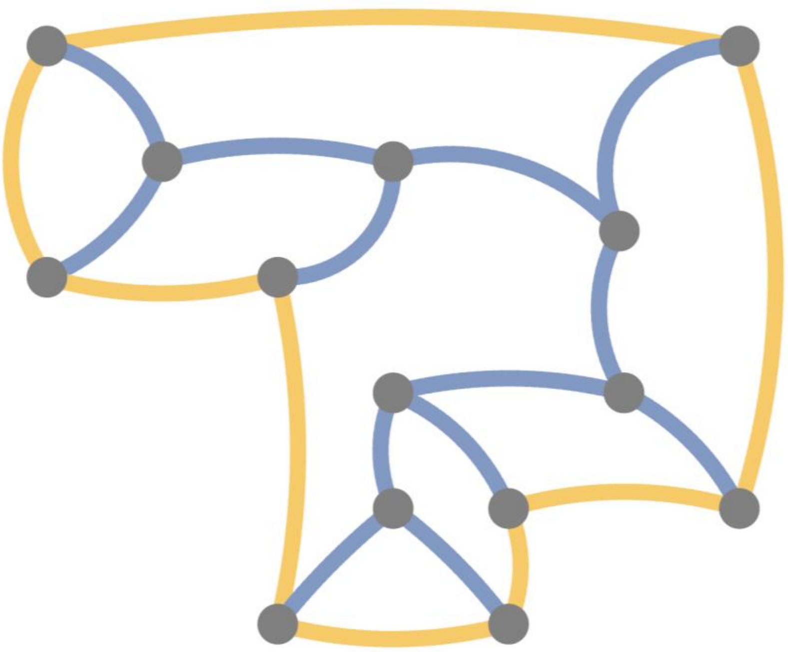


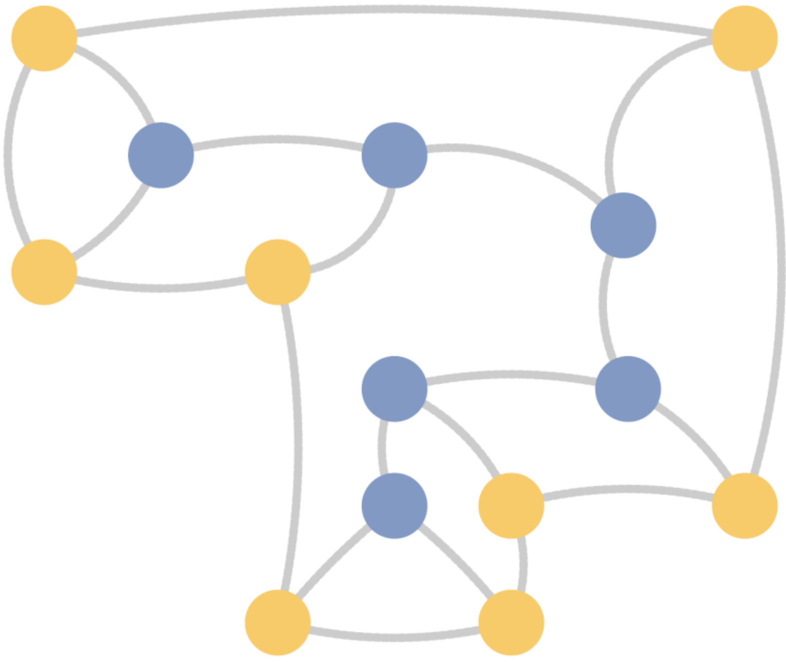
Definition 2 We call a 2D embedding of a roof graph **valid** if there exists a valid 3D embedding such that the projection of the 3D embedding in the xy plane is exactly the same as the 2D embedding.



Methodology

Notations





Roof Edge

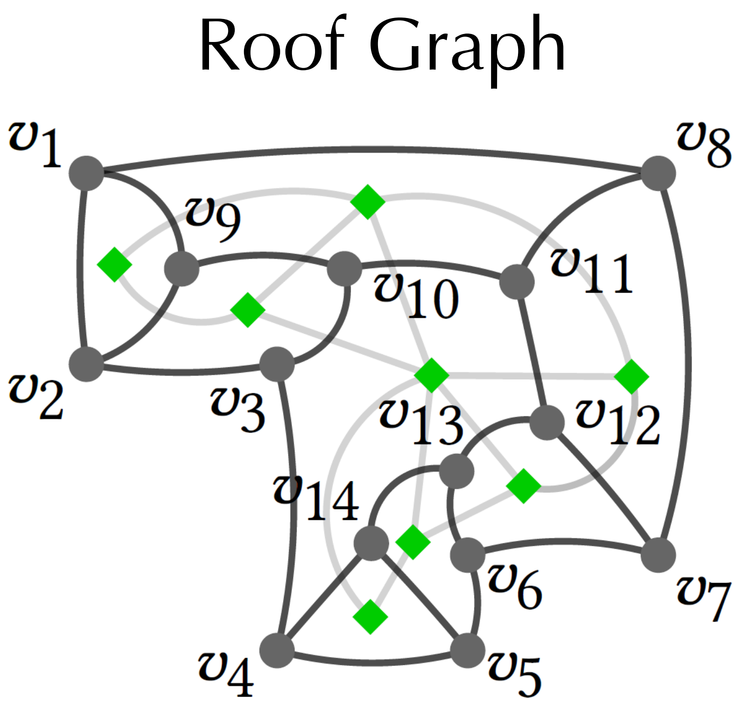
Outline Edge

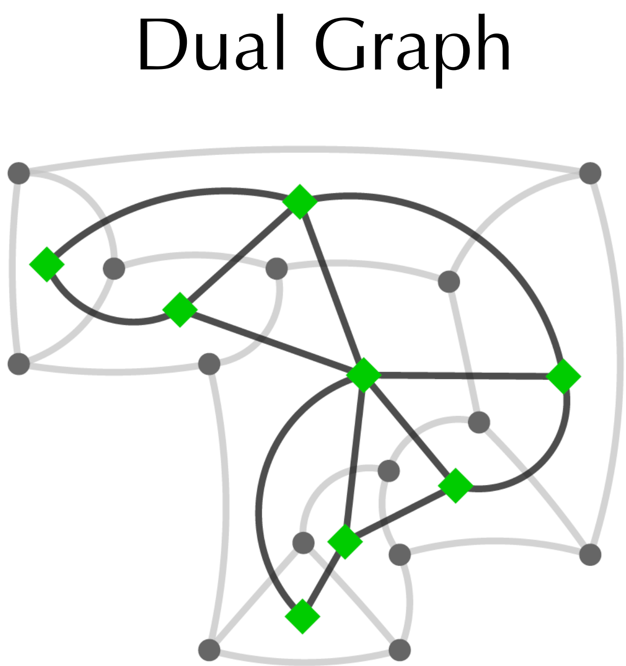
Roof Vertex

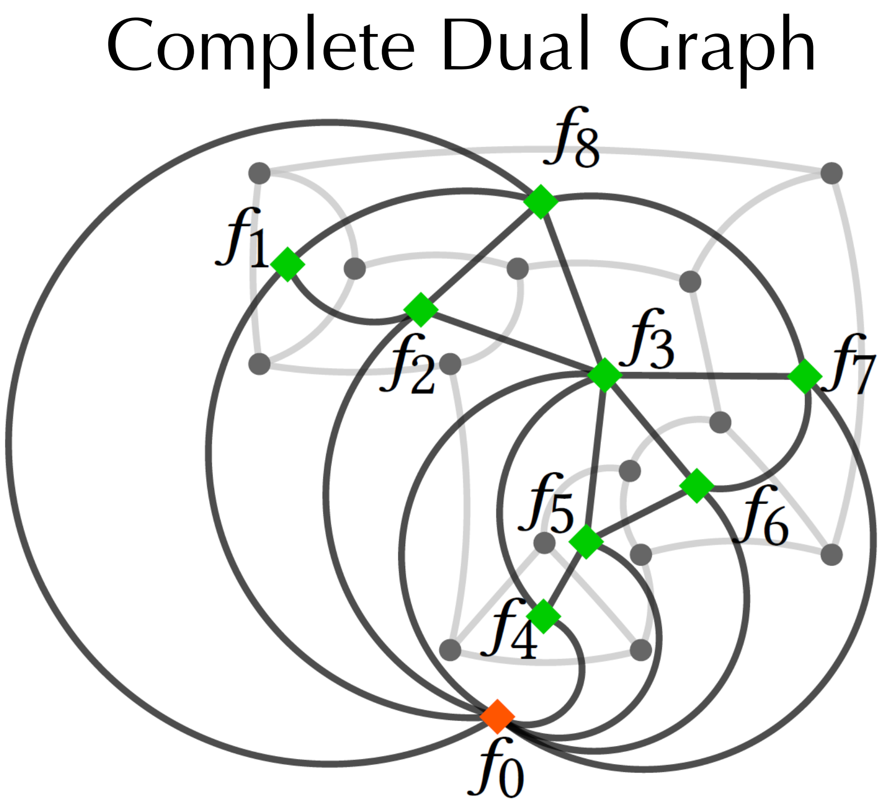
Outline Vertex

Methodology

Notations



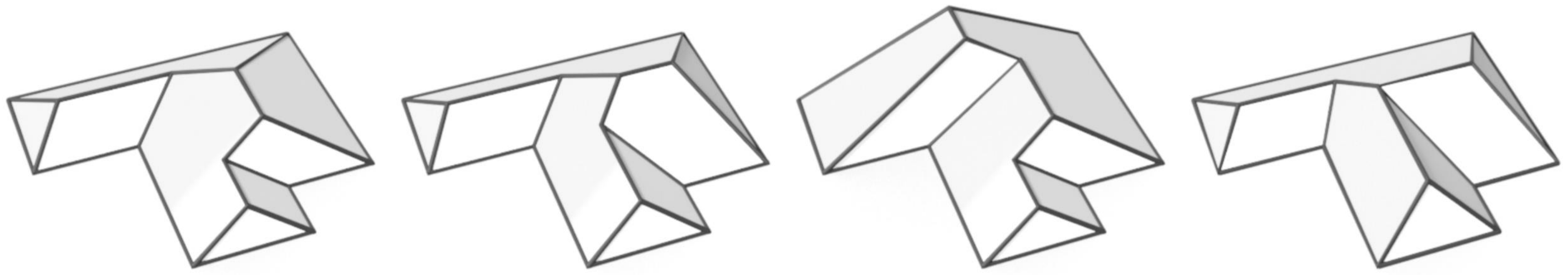


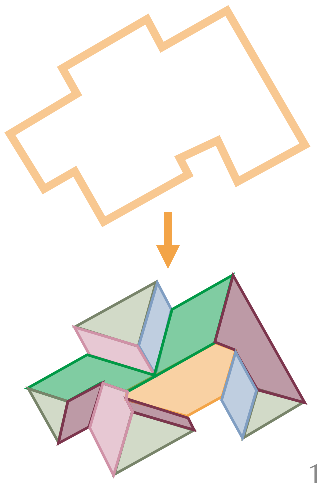


Equivalent topology representation primal graph ↔ dual graph

Methodology

Observations





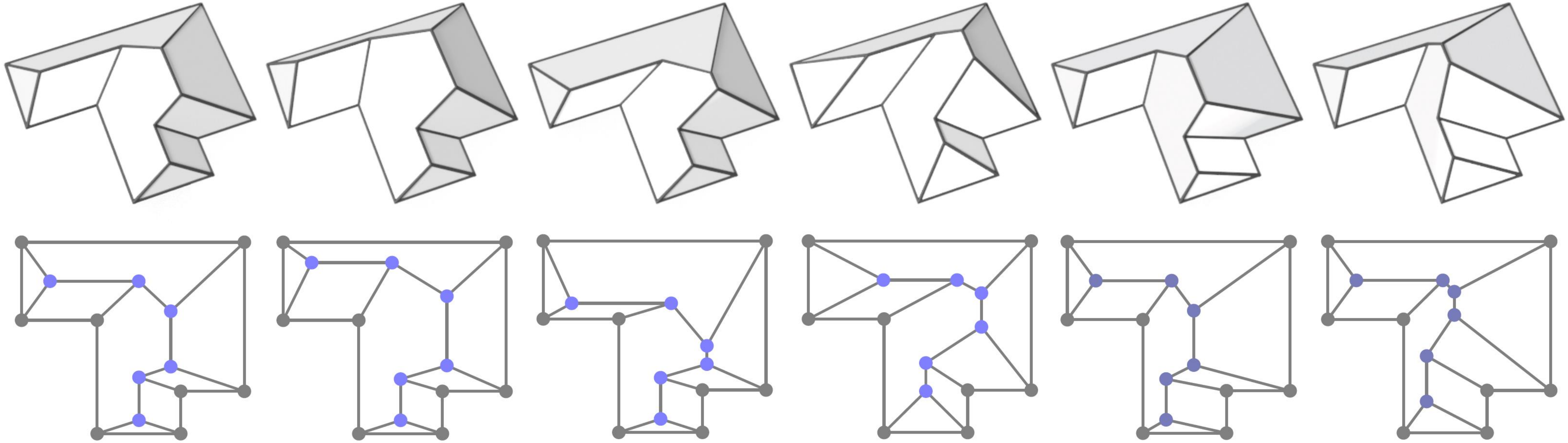
Observation roof outline → roof topology is NOT injective

- Roofs with different style (topology) can have the same outline

Solution outline + adjacency to specify the roof topology

Methodology

Observations



Observation multiple valid roof embeddings exist

- Roofs with the same topology can have different embeddings

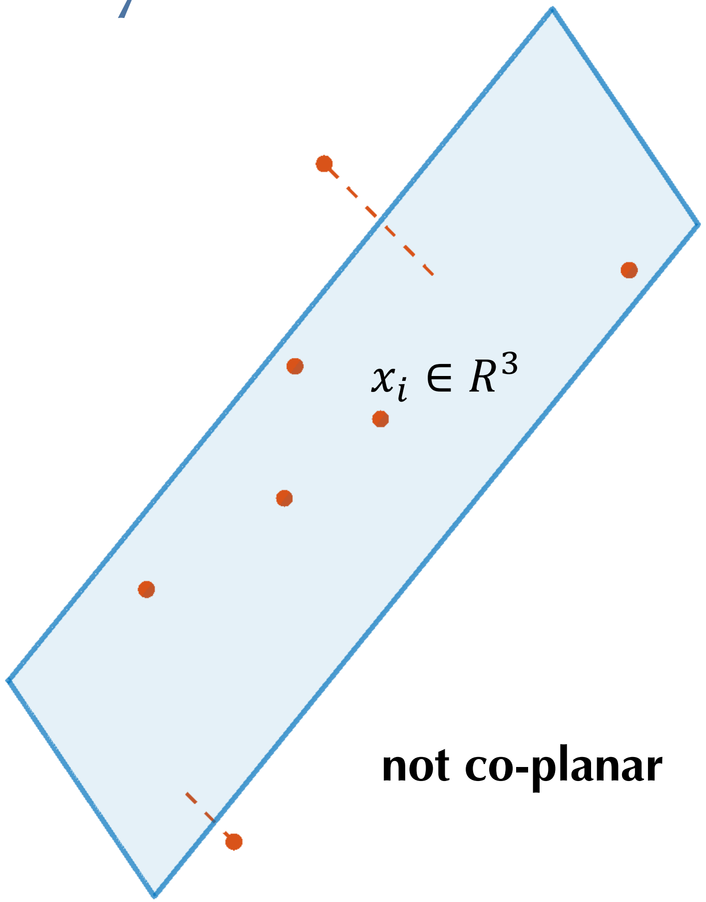
Solution regularizers to rule out undesirable embeddings

Methodology

Co-planarity Metric

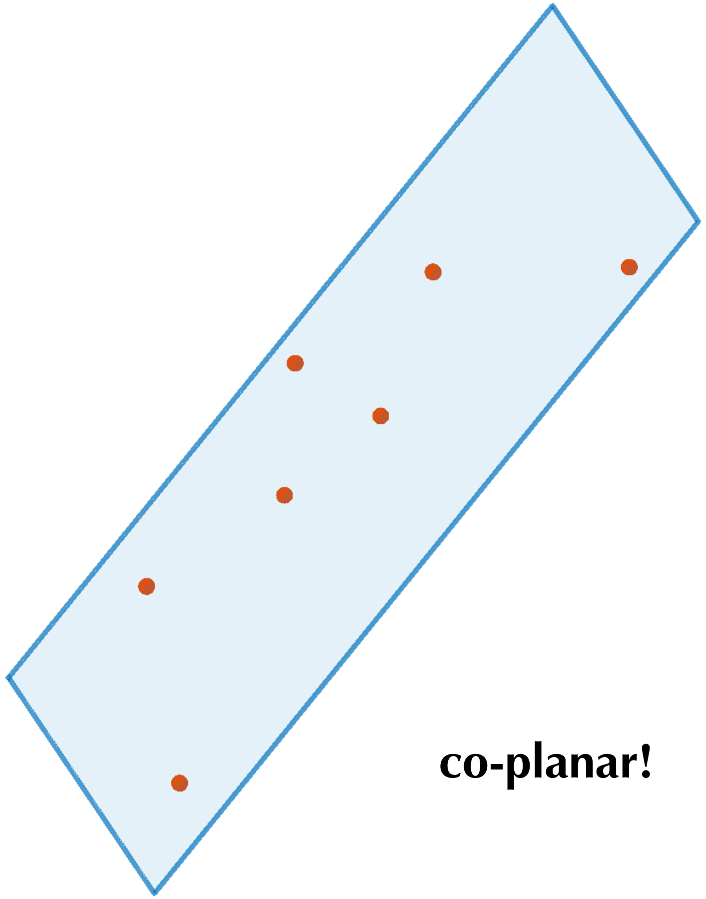
Covariance $\text{Cov}(X) = \left(X - \text{mean}(X)\right)^T \left(X - \text{mean}(X)\right)$

- **Symmetric** & **Positive semi-definite**
- **Smallest eigenvalue** ≥ 0



Covariance matrix $\text{Cov}(X)$ full rank

$\Longrightarrow \sigma_1\left(\text{Cov}(X)\right) > 0$



Covariance matrix $\text{Cov}(X)$ loses 1 DOF

$\Longrightarrow \sigma_1\left(\text{Cov}(X)\right) = 0$

Summary $\sigma_1\left(\text{Cov}(X)\right)$ measures the co-planarity error of a point set X

Methodology
Validity Formulation

- $\text{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face

Input roof outline + topology (primal/dual graph)

Output a valid 3D roof embedding

- valid: each 3D roof face is planar

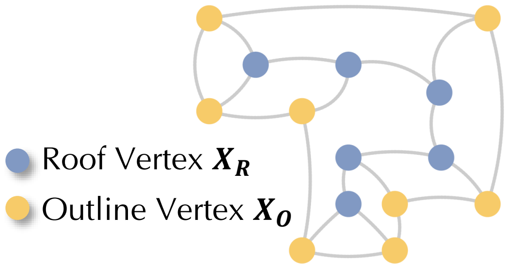
Solution optimization-based formulation

- variables: the 3D positions for each vertex
- objective: the vertices in each face are co-planar

$$\mathrm{E}_{\text{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\text{Cov}(X_{f_i})\right)$$

Methodology

Planarity Optimization



- $\mathrm{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $\mathrm{E_{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\mathrm{Cov}(X_{f_i})\right)$

$$\min_{X_R} \mathrm{E_{planarity}}(X)$$

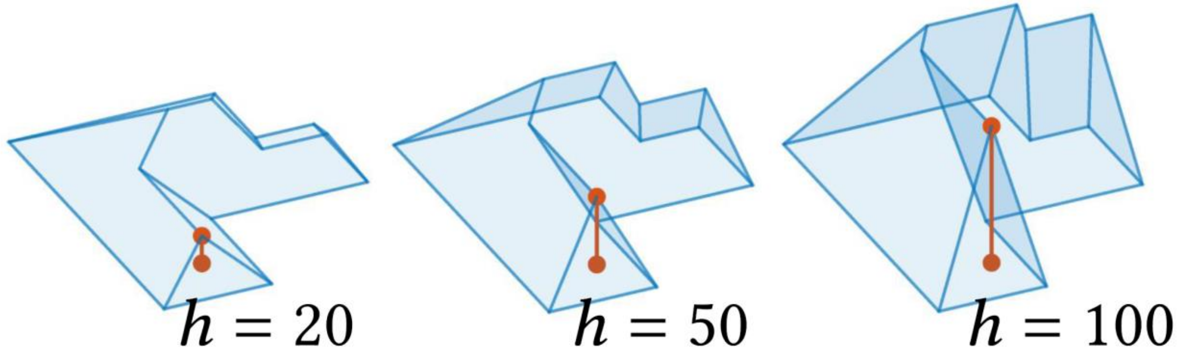
$$x_z^* = h$$

Variables 3D positions for roof vertices X_R

- Fixed outline: same setting as straight skeleton based methods

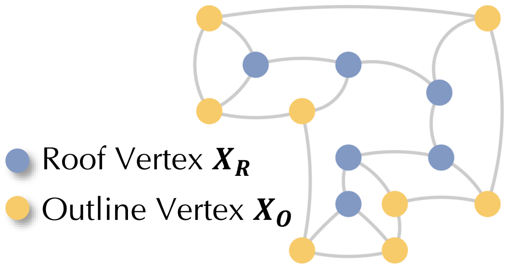
Hard constraint avoid zero-height roof

- Hyper-parameter h allows us to control the overall height of the constructed roof



Methodology

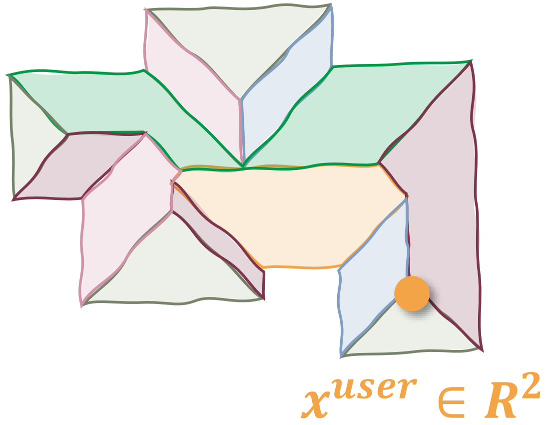
Planarity & User Input



- $\mathrm{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $\mathrm{E}_{\text{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\mathrm{Cov}(X_{f_i})\right)$
- $\bar{X}$: the 2D projection of X

$$\min_{X_R} \mathrm{E}_{\text{planarity}}(X) + \left\| \bar{X}_R - \bar{X}_R^{user} \right\|_F^2$$

$$x_z^* = h$$



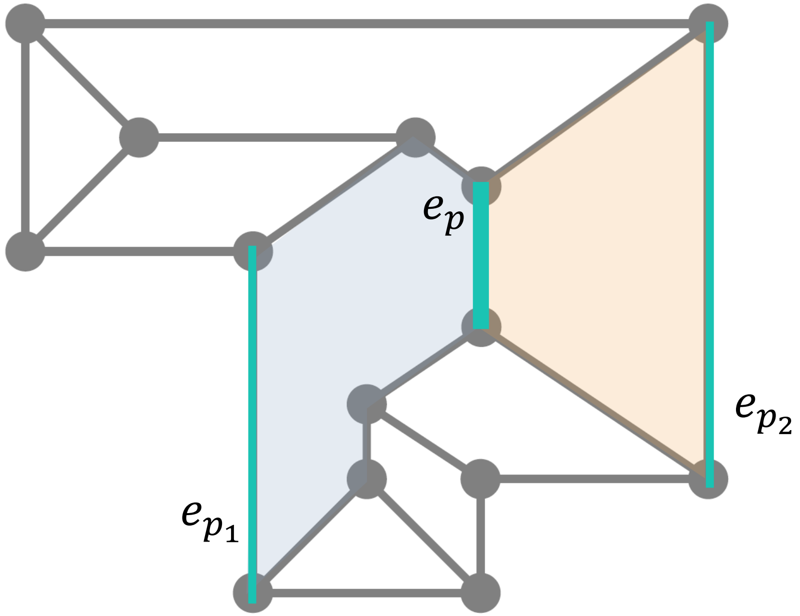
User input not valid but provides good initial locations

- enforce 2D projections of X to be close to user input

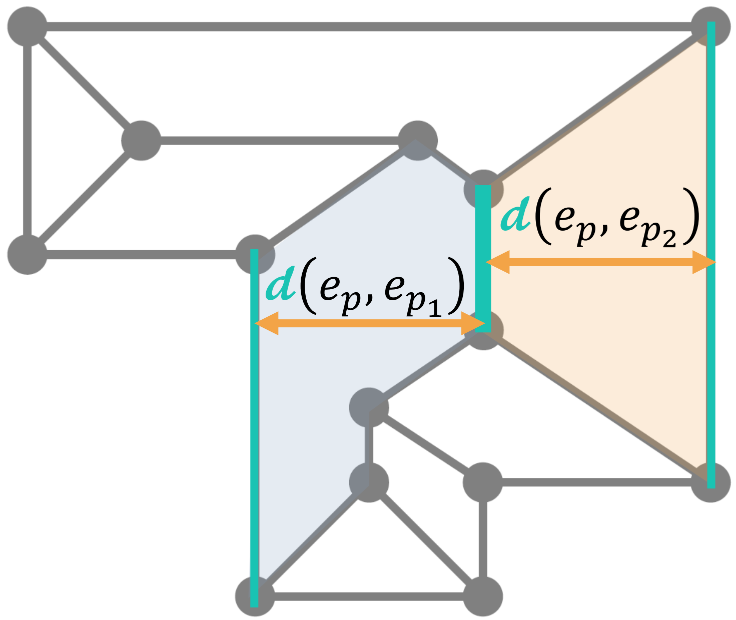
Methodology

Planarity & Aesthetic Constraints

- $\text{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $E_{\text{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\text{Cov}(X_{f_i})\right)$
- $d(e_p, e_q)$: **distance** between two parallel vectors



Parallel $e_p \parallel e_{p_1} \parallel e_{p_2}$

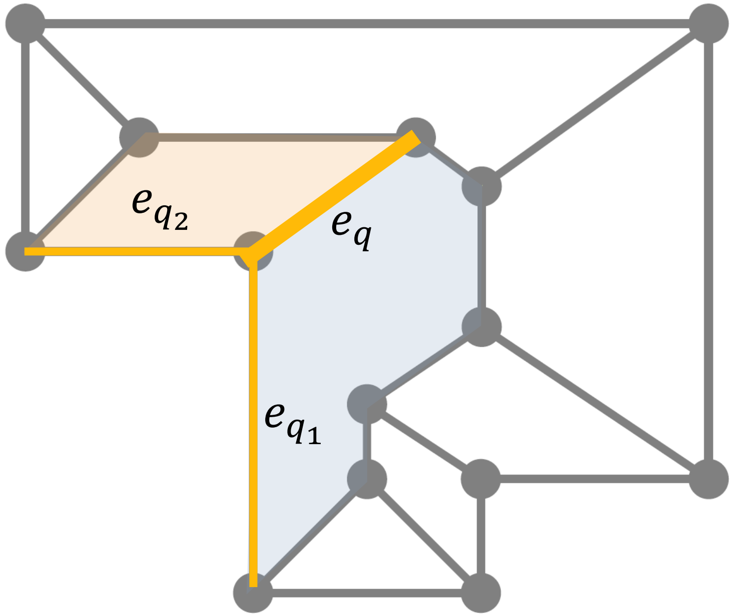


$$E = \left\| d(e_q, e_{q_1}) - d(e_q, e_{q_2}) \right\|_F^2$$

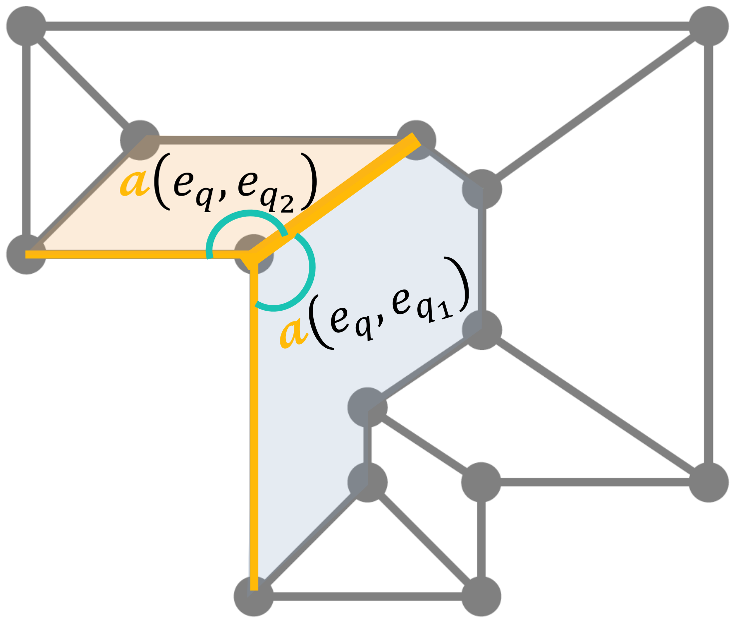
Methodology

Planarity & Aesthetic Constraints

- $\mathrm{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $\mathrm{E}_{\text{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\mathrm{Cov}(X_{f_i})\right)$
- $a(e_p, e_q)$: **angle** between two parallel vectors



Intersecting $e_q \wedge e_{q_1} \wedge e_{q_2}$



$$\mathrm{E} = \left\| a(e_q, e_{q_1}) - a(e_q, e_{q_2}) \right\|_F^2$$

Methodology

Planarity & Aesthetic Constraints

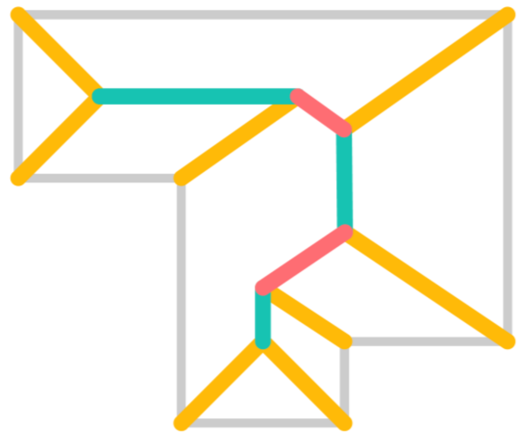
- $\mathrm{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $\mathrm{E}_{\mathrm{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\mathrm{Cov}(X_{f_i})\right)$

$$\min_{X_R} \mathrm{E}_{\mathrm{planarity}}(X) + \mathrm{E}_{aesthetic}(X)$$

$$x_z^* = h$$

$$\mathrm{E}_{\mathrm{aes.}}(X) = \sum_{p \in \{\blacksquare\}} \left\| a(e_p, e_{p_1}) - a(e_p, e_{p_2}) \right\|_F^2 + \sum_{q \in \{\blacksquare\}} \left\| d(e_q, e_{q_1}) - d(e_q, e_{q_2}) \right\|_F^2$$

- $a(e_p, e_q)$: angle between two vectors
- $d(e_p, e_q)$: distance between two parallel vectors



Aesthetic constraints more plausible local minima

- Yellow edges: angle bisectors
- Green edges: equal distance to the parallel outline edges

Methodology

Planarity & Free Outlines

- $\mathrm{Cov}(X)$: covariance matrix of X
- $\sigma_1(A)$: the smallest eigenvalue of A
- X_{f_i}: vertex positions of the f_i-th face
- $\mathrm{E}_{\text{planarity}}(X) = \sum_{i=1}^{n_f} \sigma_1\left(\mathrm{Cov}(X_{f_i})\right)$
- $\mathrm{Var}(x)$: variance of a vector x

$$\min_{x_{xyz},\, x_z} \mathrm{E}_{\text{planarity}}(X) + \eta \mathrm{Var}(x_z) + \eta \mathrm{Var}(x_z)$$

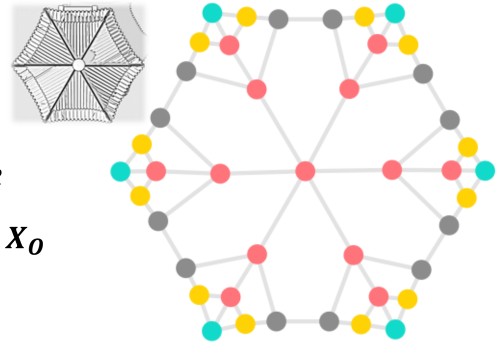
$$x_z^* = h$$

Hexagonal Pavilion

Set outline vertices as free variables

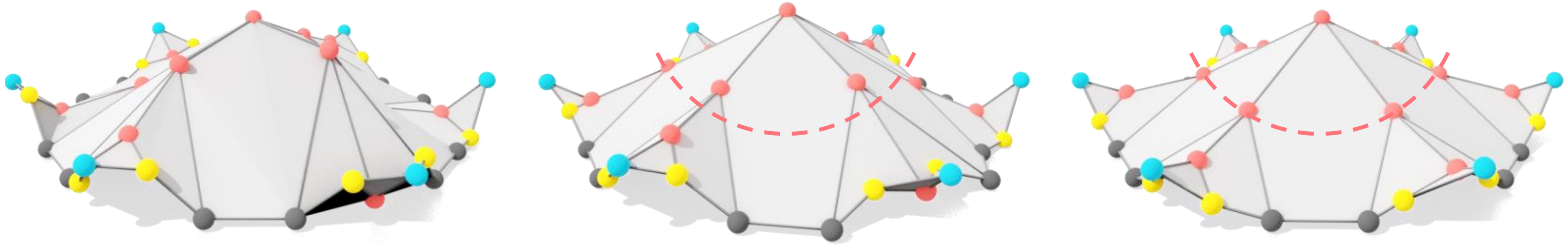
- Add extra constraints: subset of outline vertices in similar height

roof graph



Roof Vertex X_R

Outline Vertex X_O



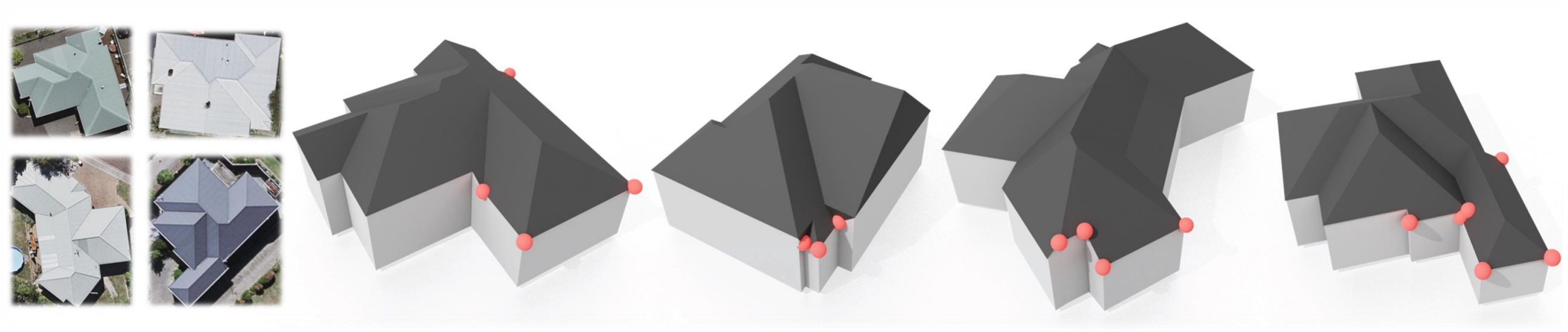
Initial embedding err $> 10^4$

Optimized ($\eta = 0$) err $< 10^{-9}$

Optimized ($\eta = 1$) err $< 10^{-9}$

Methodology

Planarity & Free Outlines

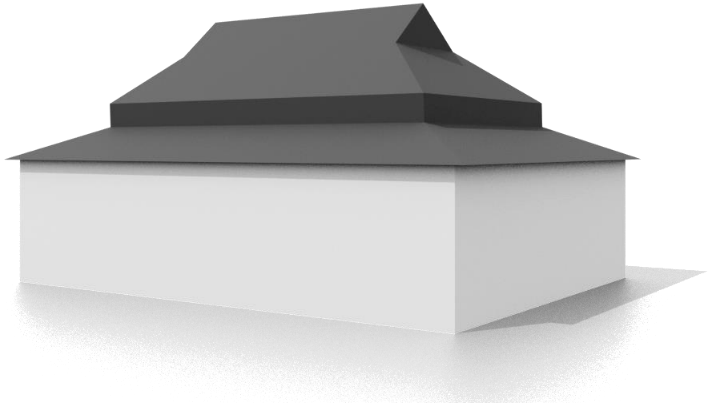


Set outline vertices as free variables

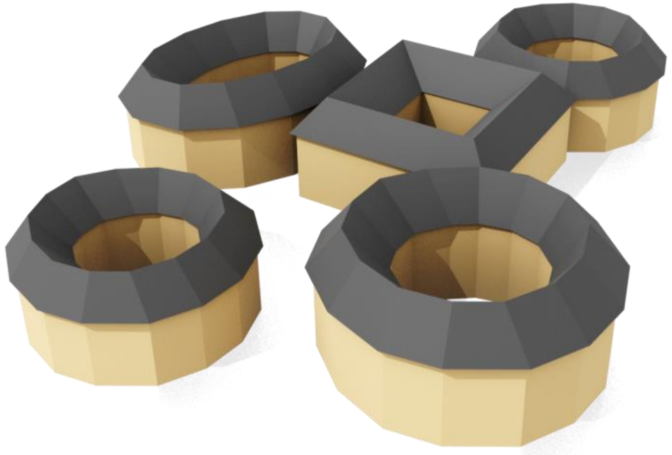
- ❖ red: outline vertices with non-zero height

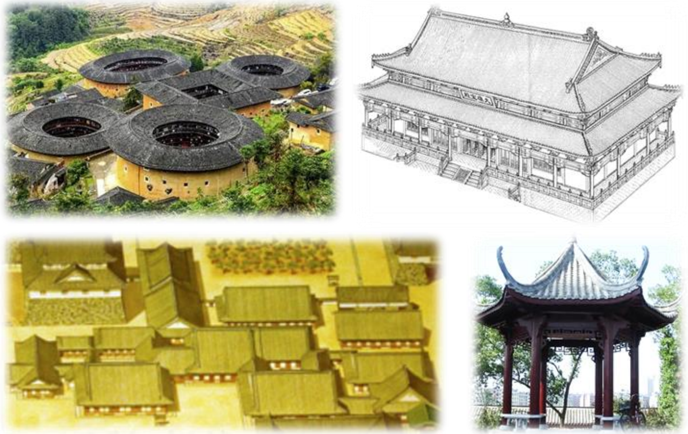
Results

Temple



Hakka Tulou, China

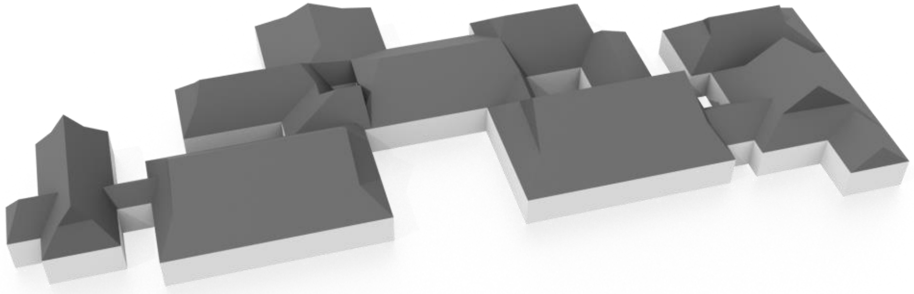




(Reference images from internet)

Hexagonal Pavilion

Nagoya Castle, Japan



Feasibility model roofs with different styles:

- ✓ Approximate curved roofs
- ✓ Roofs with vertical facades
- ✓ Roofs with inner courtyards
- ✓ Outline edges in different height

Results

Compare to Straight Skeleton Based Methods

Each Row

1. Input aerial images
2. Our reconstructed roofs (texture)
3. Our reconstructed roofs (geometry)
4. Straight Skeleton
5. Weighted Straight Skeleton (wss)

Results

Compare to Straight Skeleton Based Methods

More expressive!

No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
n_v		22	24	25	25	27	27	32	33	33	33	34	35	36	39	39	51
n_f		12	17	14	14	15	16	17	20	18	20	18	21	19	22	21	38
#err	**Ours**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	ss	0	**2**	0	0	**1**	**2**	**1**	**2**	0	**3**	**2**	**2**	**1**	**3**	**2**	**4**
	wss	0	**2**	0	0	**1**	**2**	**1**	**1**	0	**2**	**2**	**2**	**1**	**2**	**2**	**2**
$\overline{n_v}$	**Ours**	22	24	25	25	27	27	32	33	33	33	34	35	36	39	39	51
	ss	22	22	26	26	30	28	34	40	34	38	38	40	38	40	44	50
	wss	22	22	26	26	30	28	34	40	34	38	38	40	38	40	44	50
$\overline{n_f}$	**Ours**	12	17	14	14	15	16	17	20	18	20	18	21	19	22	21	38
	ss	12	12	14	14	16	15	18	21	18	20	20	21	20	21	23	26
	wss	12	12	14	14	16	15	18	21	18	20	20	21	20	21	23	26
t (s)	**Ours**	89.4	115	153	97.9	114	92.9	151	145	155	167	148	158	179	199	178	284
	ss	16	20	24	18	20	16	28	29	27	33	32	35	31	33	38	38
	wss	-	300	-	-	60	180	180	300	-	120	180	60	60	480	180	360

*GT roof **complexity***

*visual **inconsistencies***

*number of **vertices** on the constructed roofs*

*number of **faces** on the constructed roofs*

runtime

Baselines

- Straight Skeleton (ss)
- Weighted Straight Skeleton (wss)

Results

Compare to Commercial Software

3ds Max allows non-planar faces!

Manually triangulate faces to enforce planarity in SU!

More efficient

	No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
valid	**Ours**	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	3D	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
	SU	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
#err	**Ours**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3D	9	12	10	11	12	12	16	15	15	11	16	15	16	18	17	20
	SU	20	16	21	19	16	27	33	21	25	38	28	30	33	33	29	42
poly (%)	**Ours**	75	65	71	79	67	69	82	75	83	60	72	57	79	73	71	42
	3D	75	62	71	79	75	67	87	75	83	52	76	60	80	72	74	56
	SU	0	24	2.6	3.3	20	2.3	2.0	22	7.0	10	11	9.8	3.8	15	14	0
$\overline{n_v}$	**Ours**	22	24	25	25	27	27	32	33	33	33	34	35	36	39	39	51
	3D	22	31	37	26	28	28	44	34	34	35	44	35	36	40	39	53
	SU	22	29	26	25	28	30	36	45	33	52	39	40	37	46	42	58
$\overline{n_f}$	**Ours**	12	17	14	14	15	16	17	20	18	20	18	21	19	22	21	38
	3D	12	21	14	14	16	18	18	20	18	21	21	25	20	25	23	36
	SU	32	33	35	33	31	43	50	41	43	58	46	51	52	55	50	82
t (min)	**Ours**	1.5	1.9	2.6	1.6	1.9	1.6	2.5	2.4	2.6	2.8	2.5	2.6	3.0	3.3	3.0	4.7
	3D	6	7	7	6	6	12	12	11	7	14	6	9	10	8	10	23
	SU	12	16	15	12	22	20	21	22	21	25	19	32	22	14	25	36

*Is roof **planar**?*

*Topological **errors***

*Ratio of the **polygon** faces in the roof*

*number of **vertices** on the constructed roofs*

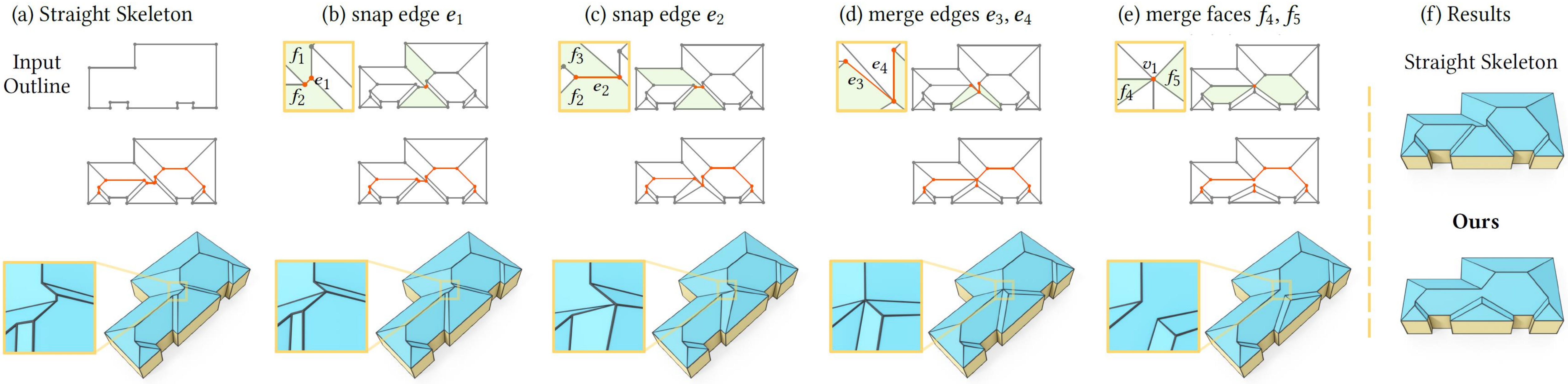
*number of **faces** on the constructed roofs*

runtime

Baselines

- 3ds Max (3D)
- SketchUp (SU)

Applications
Interactive Editing



Editing Operations optimization-based formulation allows interactive editing

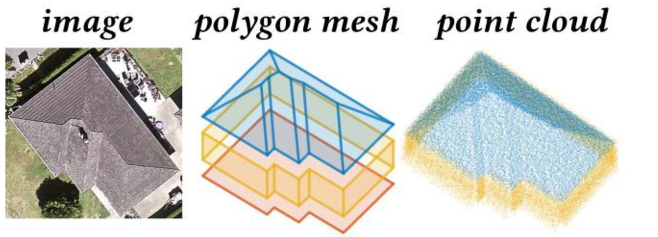
- ✓ Move a vertex
- ✓ Move an edge
- ✓ Snap an edge
- ✓ Merge two faces
- ✓ Split a face
- ✓ Force two faces to be adjacent

Applications

Roof-Image Paired Dataset

Dataset polygonal roof meshes paired with images

- ✓ >3K .obj
- ✓ Texture coordinates
- ✓ Face labels
- ✓ Roof synthesis
- ✓ Roof segmentation
- ✓ Roof detection



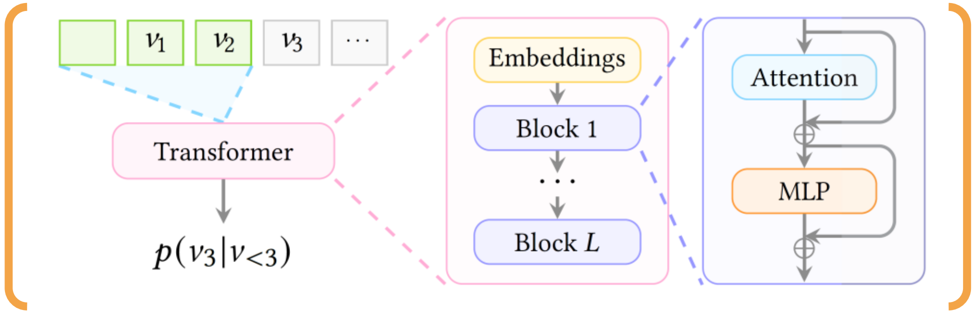
https://github.com/llorz/SGA21_roofOptimization/tree/main/RoofGraphDataset

Applications

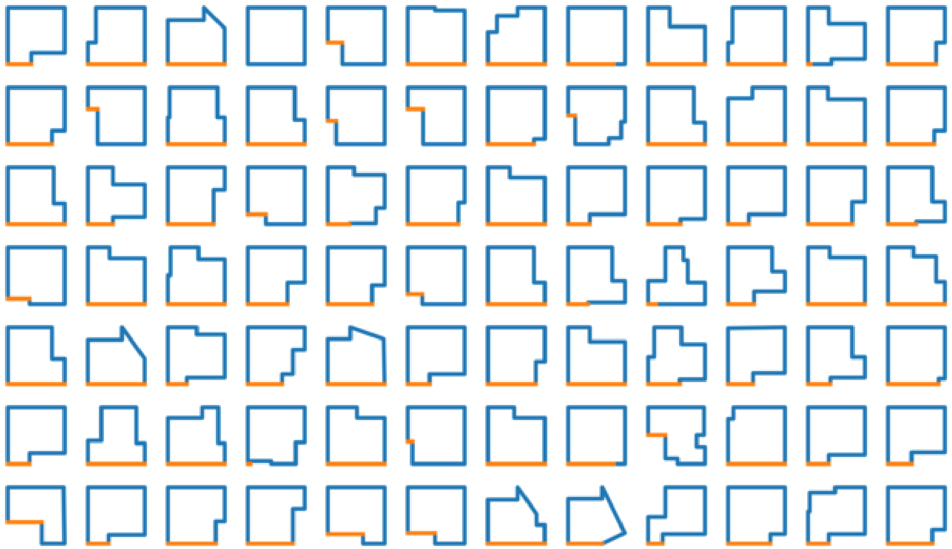
Roof Synthesis from Scratch

Training set: 2105 samples, Testing set: 210 samples

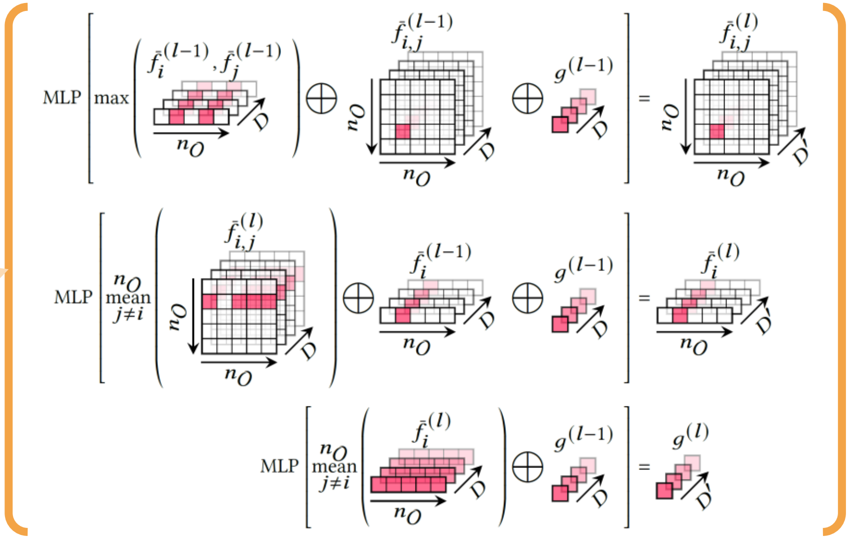
Roof Outline Generation



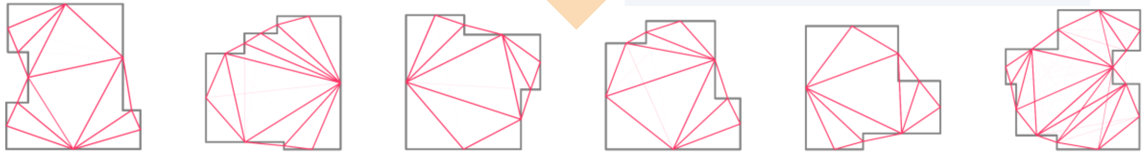
Transformer



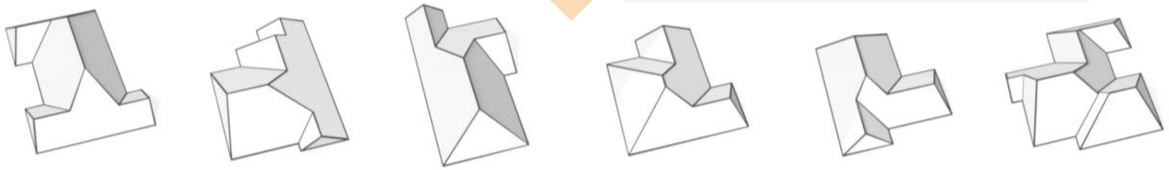
Face Adjacency Prediction



GCNs

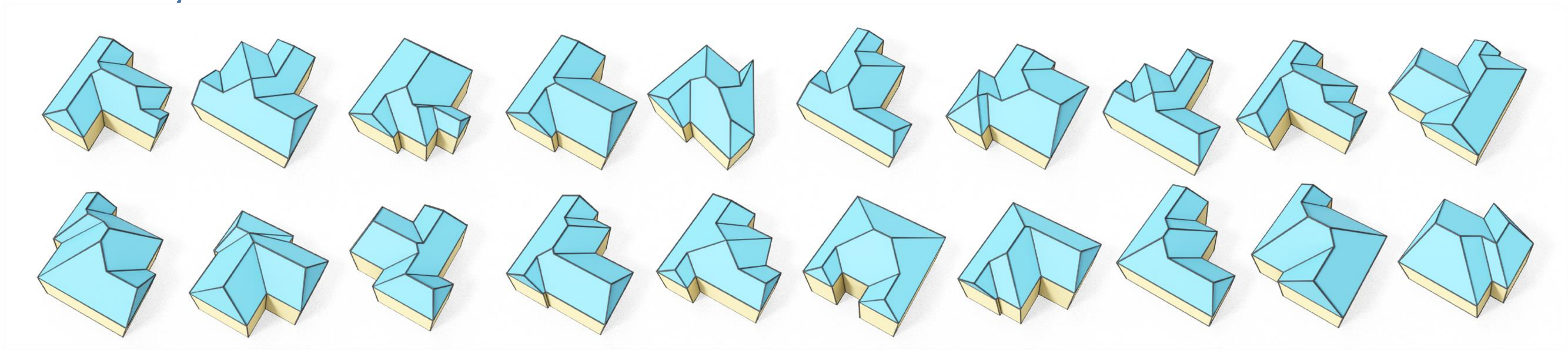


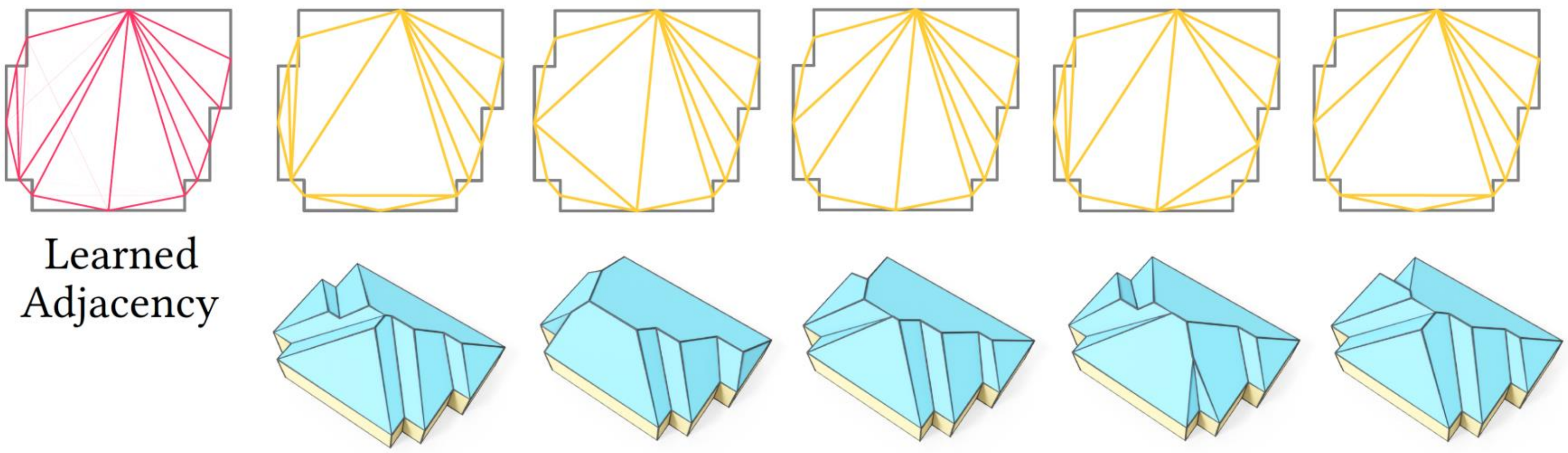
Roof Optimization



Applications

Roof Synthesis from Scratch





- ✓ Synthesize roofs from scratch
- ✓ Synthesize roofs with different styles with the same outline

Summary

Goal roof modeling + roof embedding

Baselines commercial software & straight skeleton based methods

Our Solution roof graph representation + optimization-based construction

- Efficient & Flexible
- Interactive editing
- Image-Roof paired dataset
- Automatic roof synthesis: outline generation + adjacency prediction

Limitations & Future Work

Limitations

- Cannot directly handle curved roofs including stadiums and skyscrapers
- Did not model roof textures
- Did not touch on automatic reconstruction from images

Future Work

- End-to-end roof reconstruction from images
- Practical constraints for roof fabricability
- Roof texture synthesis
- ……

Intuitive and Efficient Roof Modeling for Reconstruction and Synthesis

Jing Ren[1,2], Biao Zhang[1], Bojian Wu[2], Jianqiang Huang[2], Lubin Fan[2], Maks Ovsjanikov[3], Peter Wonka[1]

[1]KAUST, [2]Alibaba Group, [3]Ecole Polytechnique, IP Paris

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Code & Data https://github.com/llorz/SGA21_roofOptimization

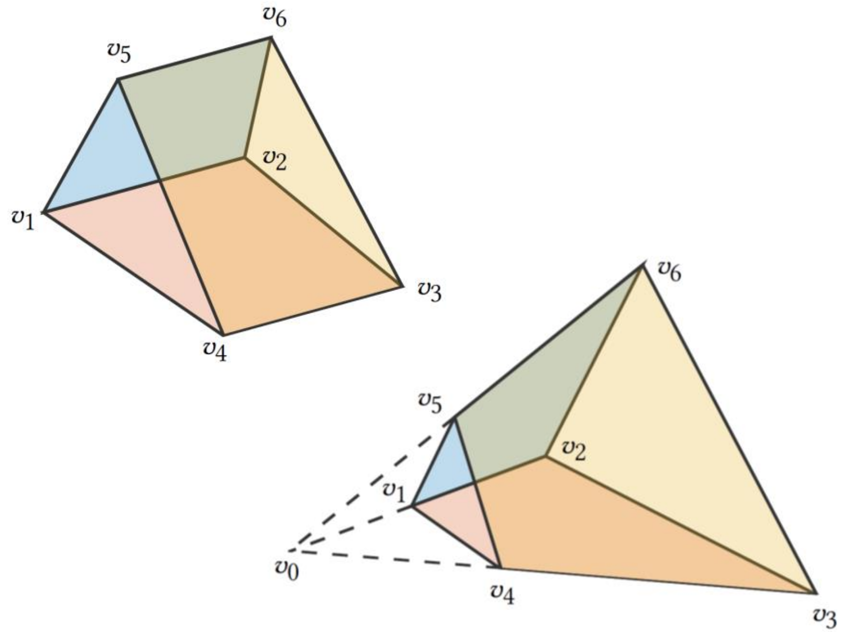
erc

Supplementary materials

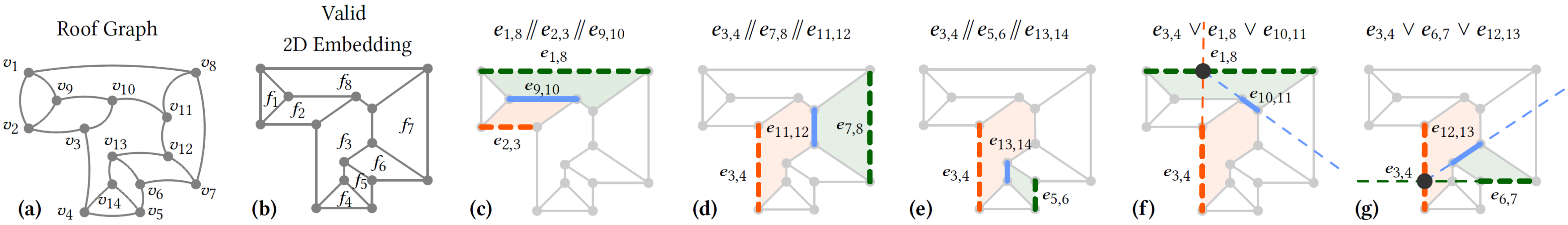
Introduction

Notations

REMARK 1 The intersecting line of two adjacent 3D planar faces with fixed outline edges, is either <u>parallel to both outline edges</u>, or <u>intersects the two outline edges at the same point</u>. The same conclusion holds when we project the 3D planar faces to *xy*-plane



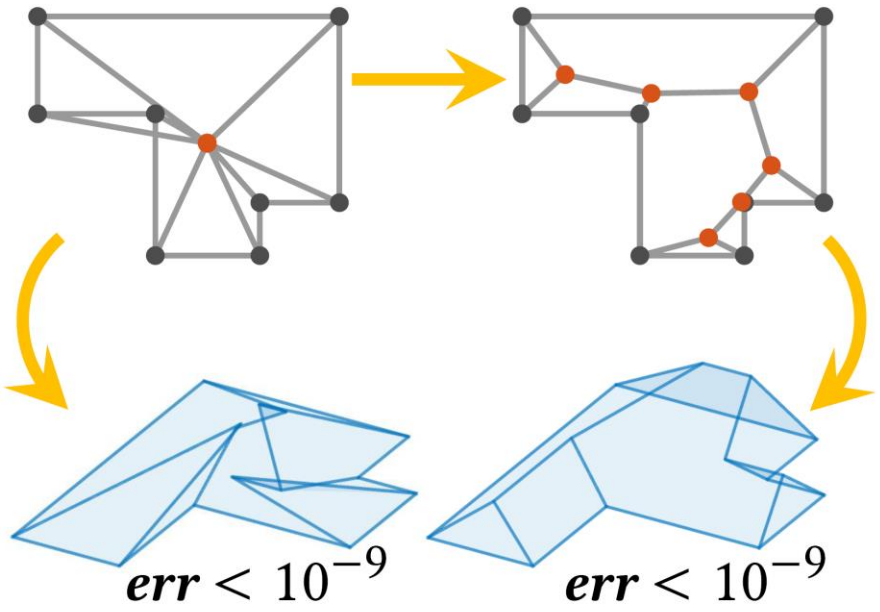
Use Remark 1 to check if a 2D embedding is valid or not



Methodology
Spectral Initialization

- $\bar{X}_O$: 2D position of the outline vertices
- $\bar{X}_R$: 2D position of the roof vertices
- A_V: vertex adjacency matrix of the roof graph, $A_V(i,j) = 1$ iff $v_i \sim v_j$
- $\mathcal{L}_V$: graph Laplacian of the roof graph, $\mathcal{L}_V = \mathrm{diag}(\mathbf{1}^T A_V) - A_V$

$$\min_{\bar{X}_R} \left\| \begin{pmatrix} \bar{X}_O \\ \bar{X}_R \end{pmatrix}^T \mathcal{L}_V \begin{pmatrix} \bar{X}_O \\ \bar{X}_R \end{pmatrix} \right\|_F^2$$

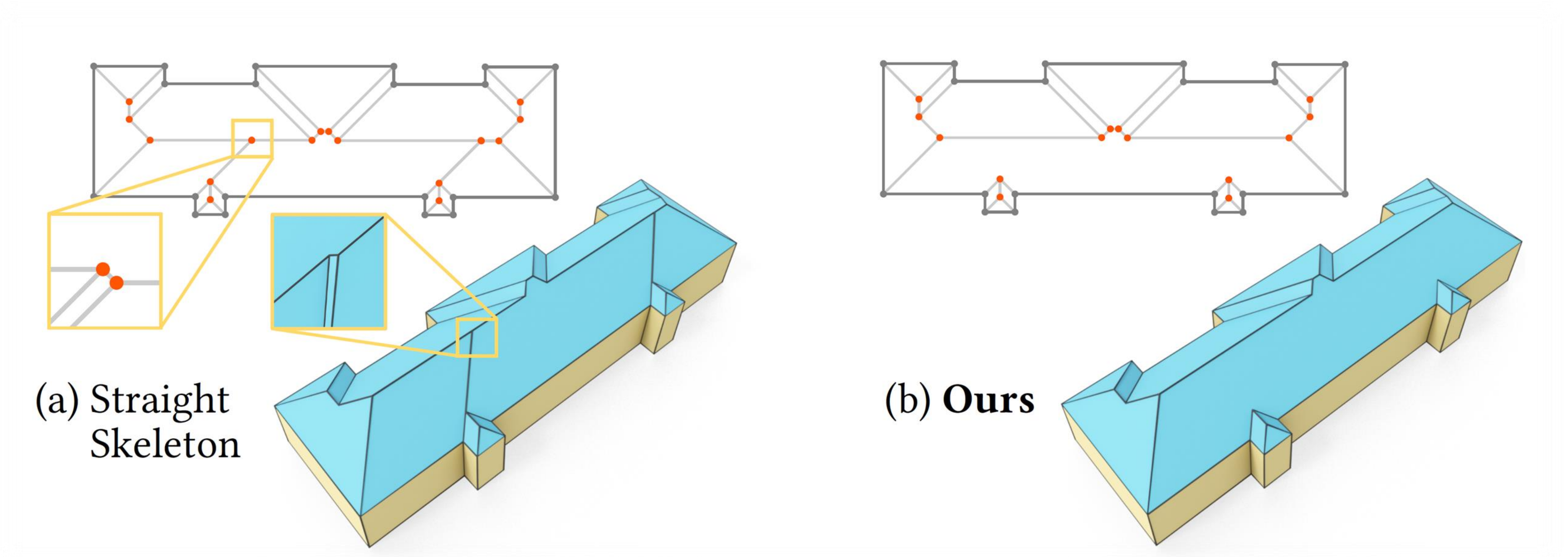


No user input as initial embedding

- **Random/Zero initialization** can lead to undesirable global minima
- **Spectral initialization** can avoid self-intersections

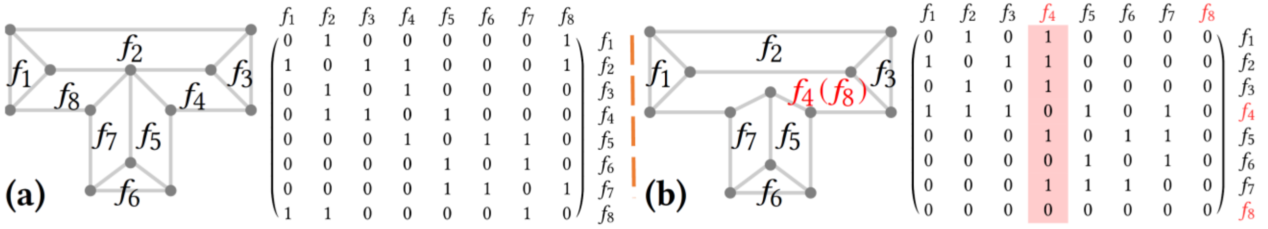
Results

Face with Multiple Outline Edges



More expressive

- ❖ Primal graph: can model this case directly
- ❖ Dual graph: the face adjacency matrix can be modified to model this case

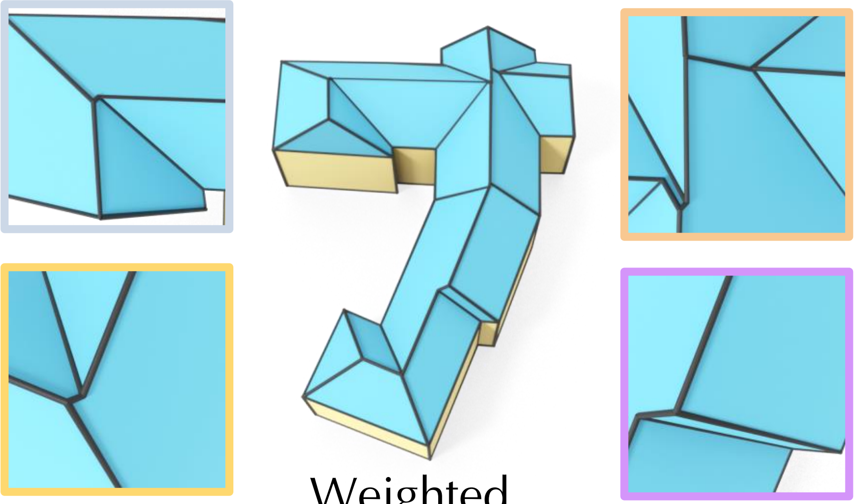


Applications

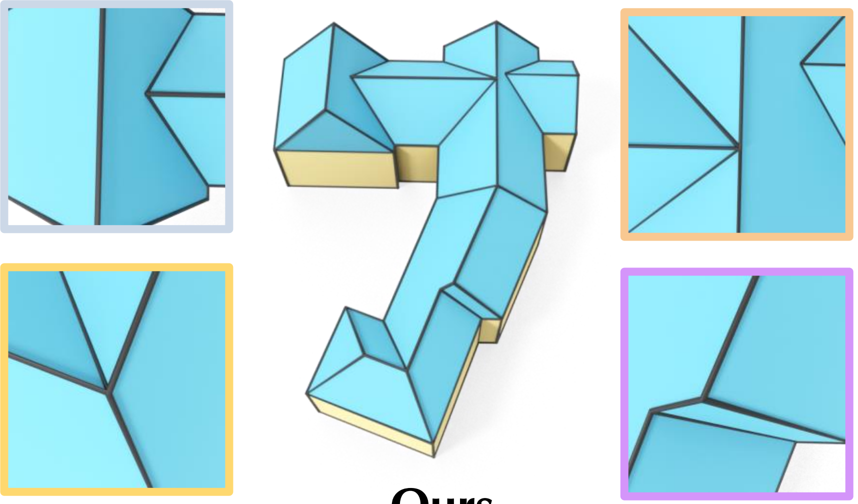
Interactive Editing

Input image

Straight Skeleton



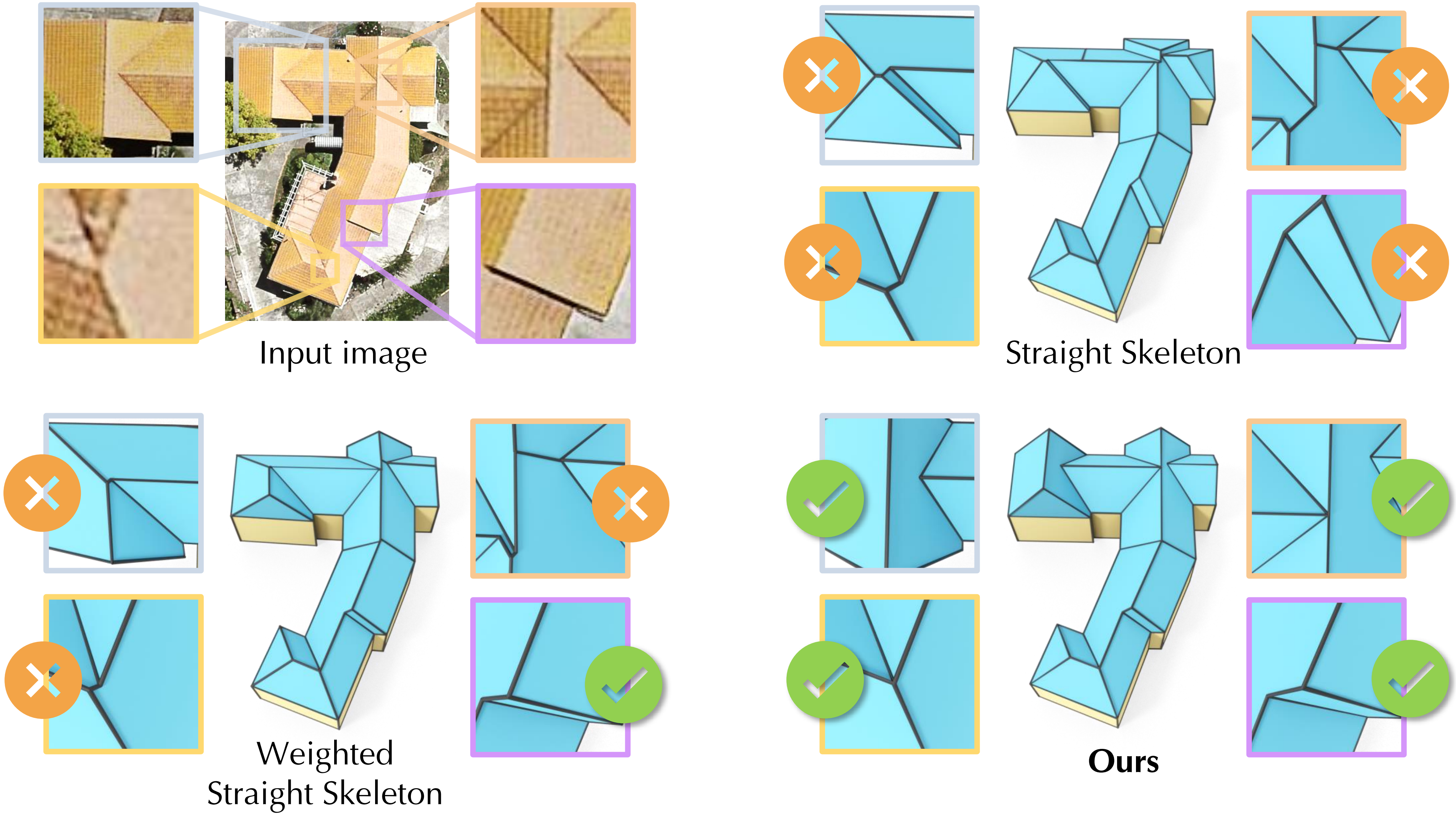
Weighted
Straight Skeleton



Ours

Applications

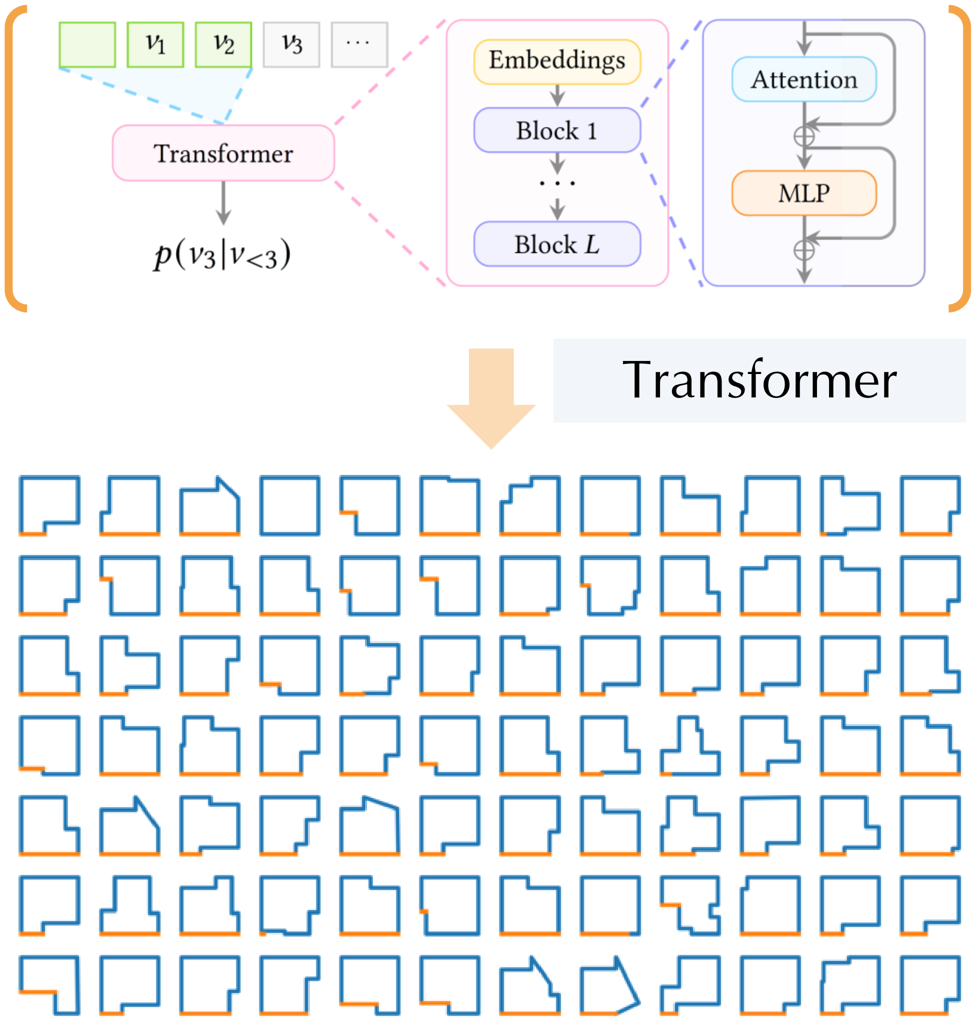
Interactive Editing



Applications

Roof Synthesis from Scratch

Roof Outline Generation



Input nothing

Output a 2D outline as a sequence of vertices $\{v_1, v_2, \cdots, v_n\}$

Tokenization (flattened) vertex position values belong to $\{1, 2, \cdots, 2^b\}$

Architectures

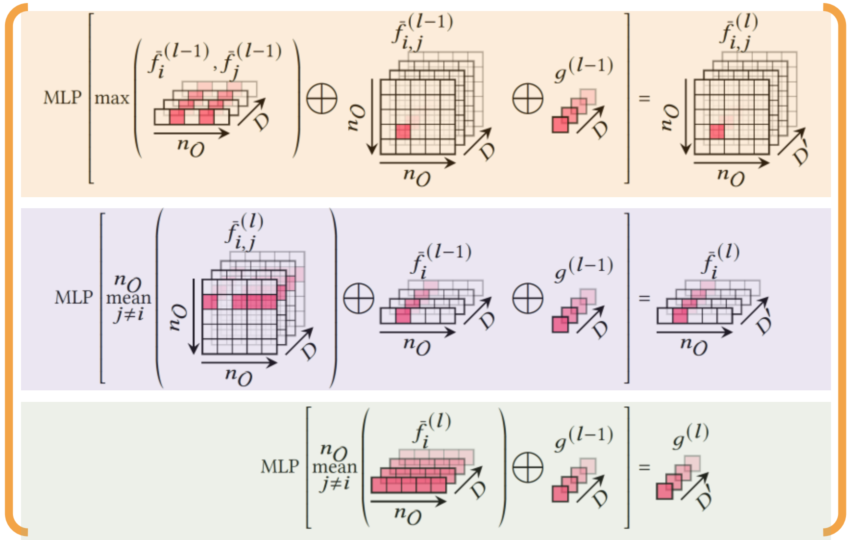
- 6 blocks: self-attention + MLP
- Embedding dimension = 384
- Self-attention: 12 heads
- MLPs: hidden dimension =1536

token:	v_1	v_2	v_3	v_4	$\cdots$	v_{2n_O-1}	v_{2n_O}	s
position:	1	1	2	2	$\cdots$	n_O	n_O	n_O+1
coord:	1	2	1	2	$\cdots$	1	2	1

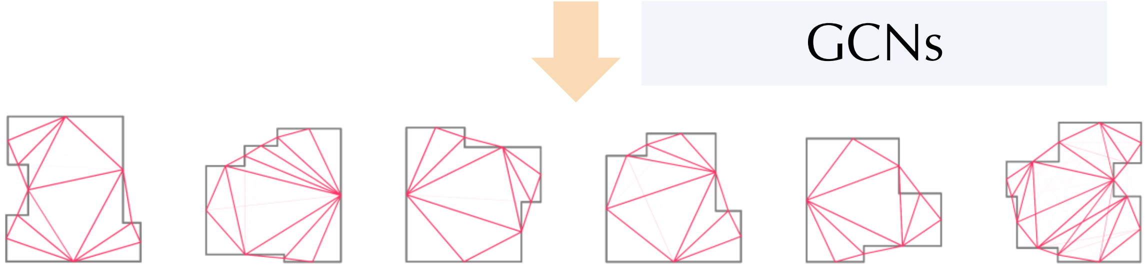
Applications

Roof Synthesis from Scratch

Face Adjacency Prediction



GCNs



Input outline $\{v_1, v_2, \cdots, v_n\}$

Output probability p_{ij} of the face f_i being adjacent to the face f_j

Architectures we stack following basic building blocks 4 times

- Adjacency block
- Edge block
- Global block

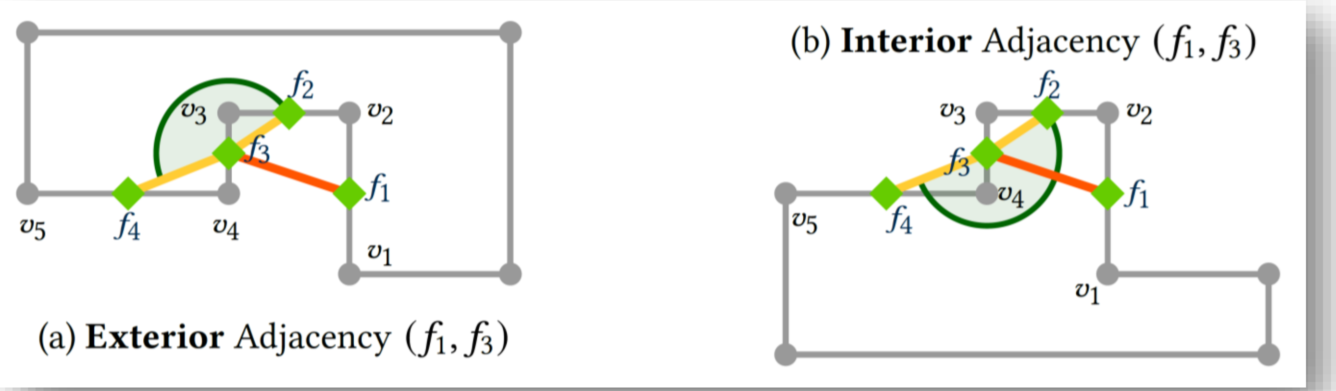
Each block updates representation vectors (adjacency, edge, global).

Loss objective binary cross entropy

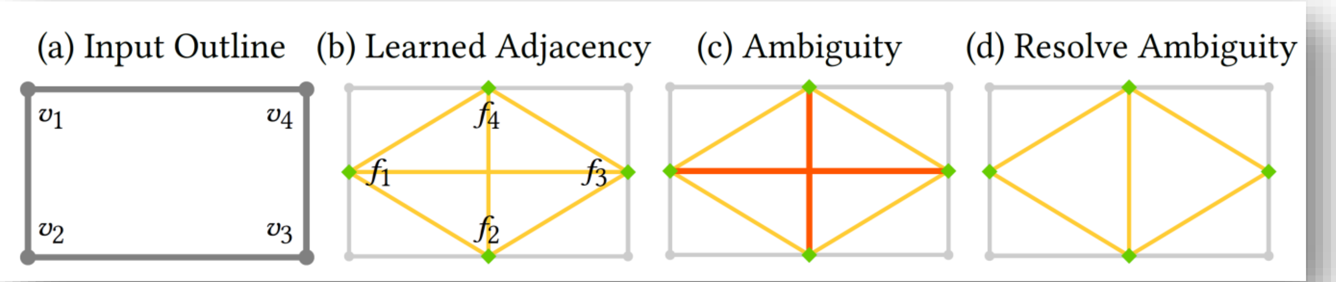
Applications

Roof Synthesis (Resolve Ambiguities in Predicted Adjacencies)

Case 01 interior vs. exterior region



Case 02 conflicting adjacencies

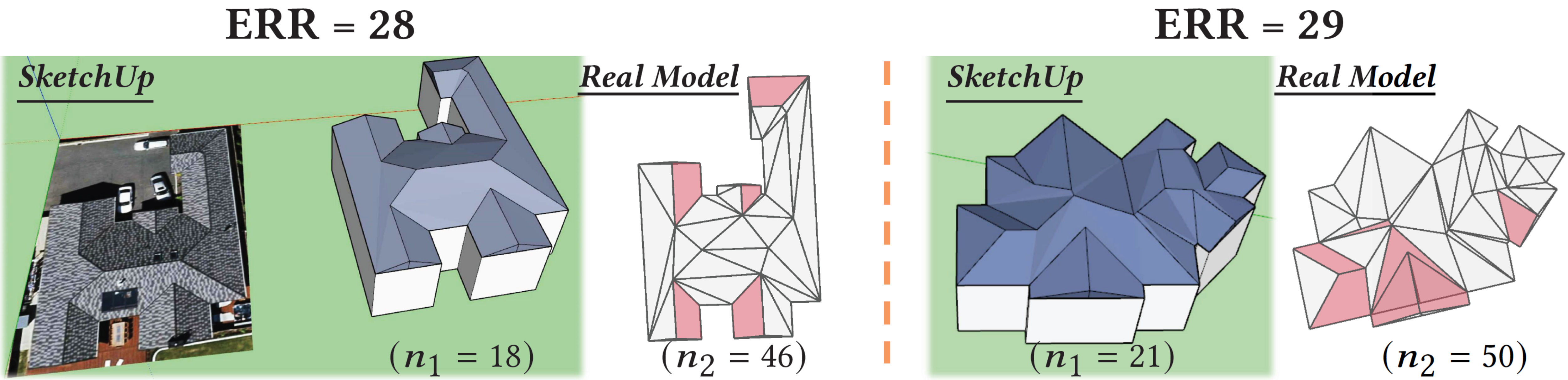


Algorithm extract valid dual graph

1. Resolve ambiguity 01: check if each edge in the dual graph (adjacency) lies in the **interior** of the roof
2. Resolve ambiguity 02: detect **edge intersections** in the dual graph, then remove the confliction by
   - ❖ Greedy (most likely one)
   - ❖ Sampling (all possible ones)

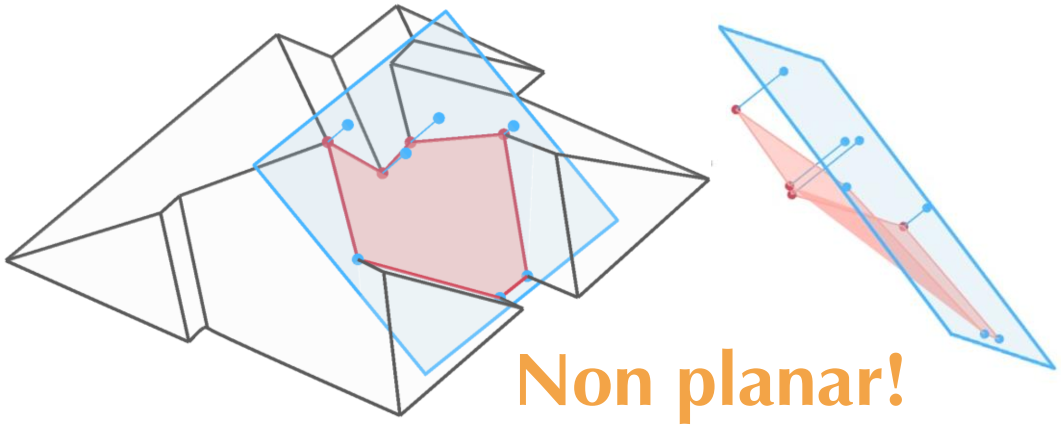
Results

Evaluate SketchUp

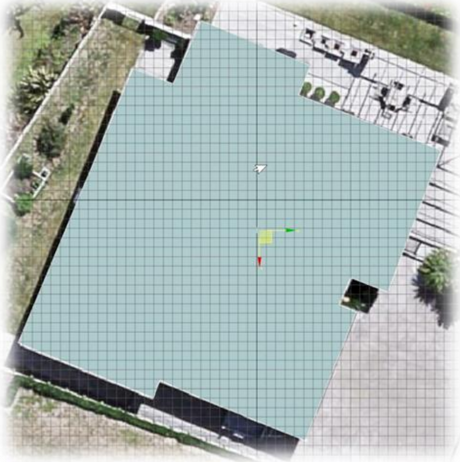


- The artist hid some edges to make the constructed roof visually consistent with the input image. n_1 reports the number of faces that are visible
- n_2 reports the actual number of faces that were created
- $\text{ERR} = n_2 - n_1$
- Red: polygonal faces in the created roof mesh

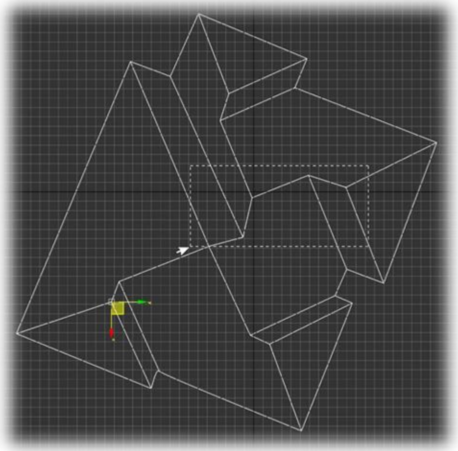
Existing Methods
Commercial Software: 3ds Max



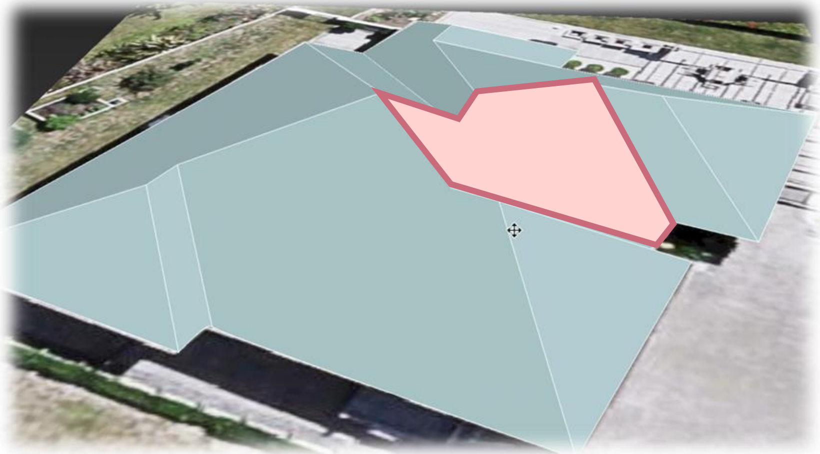
Input



- Add vertex
- Add face



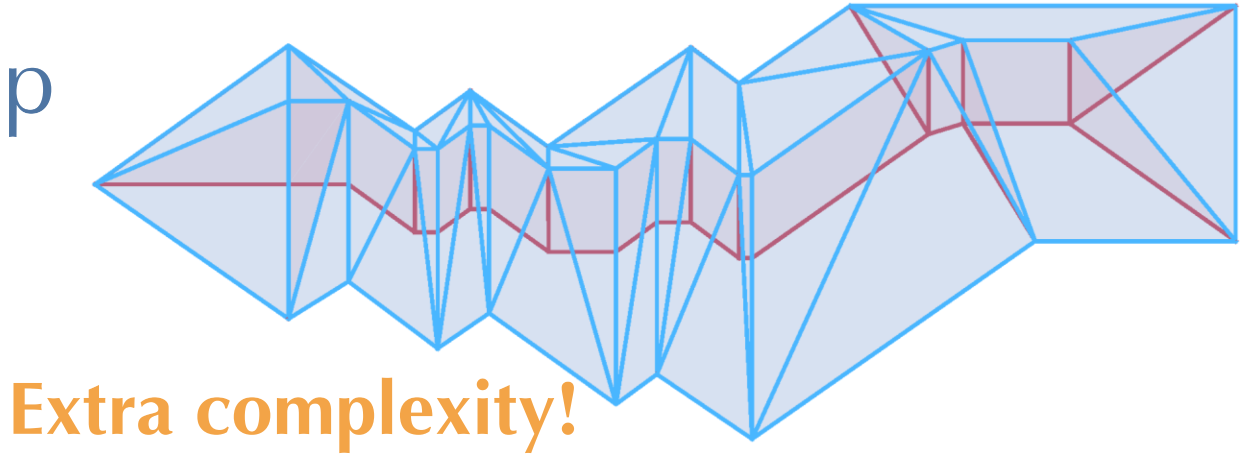
- Cut faces
- Move vertex



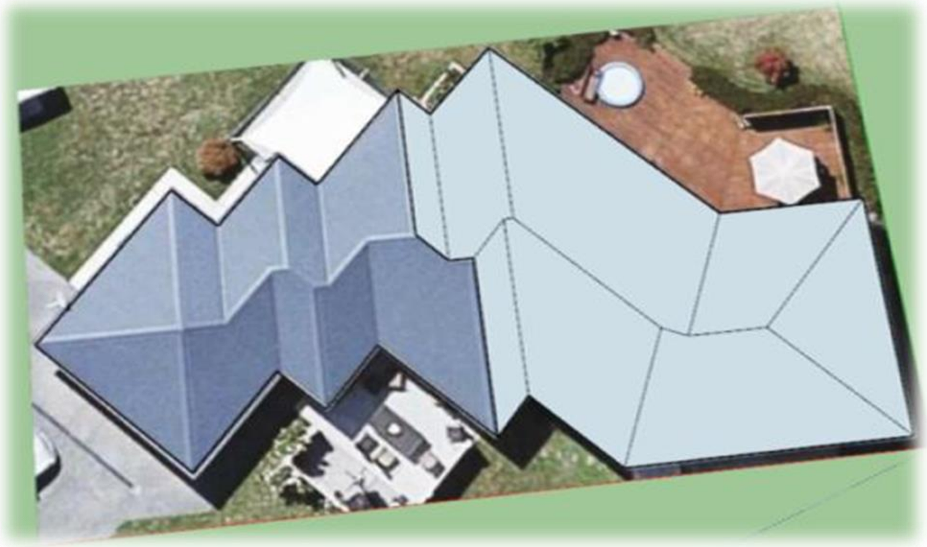
- Move vertex

Existing Methods

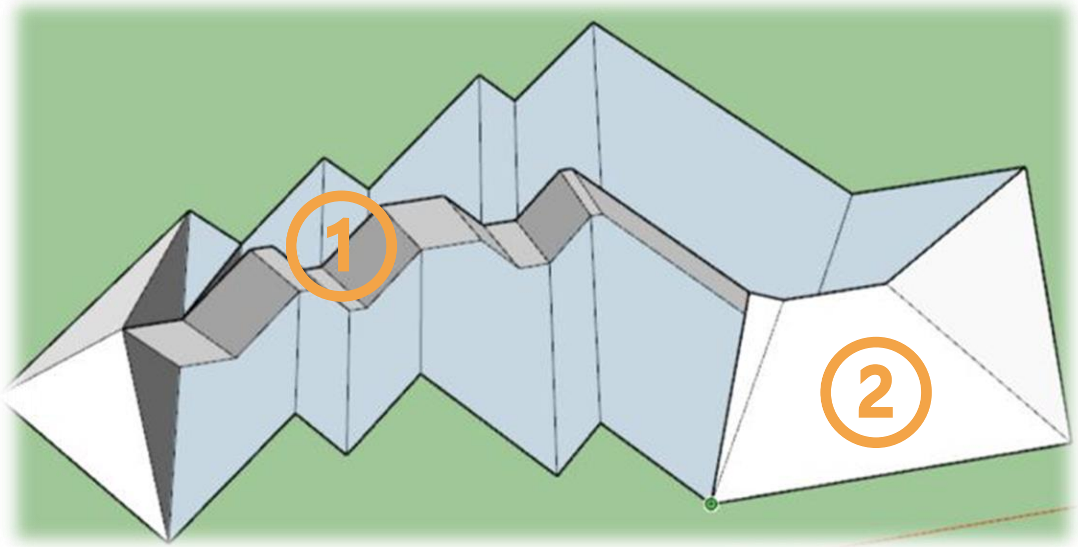
Commercial Software: SketchUp



Input



Specify roof topology



1.Build roof beams
2.Add planar roof tops